Eur. Phys. J. A **26**, 125–134 (2005)
DOI 10.1140/epja/i2005-10140-1

THE EUROPEAN PHYSICAL JOURNAL A

Covariant tensor formalism for partial-wave analyses of ψ decays into $\gamma B\bar{B}$, $\gamma\gamma V$ and $\psi(2s) \to \gamma\chi_{c0,1,2}$ with $\chi_{c0,1,2} \to K\bar{K}\pi^+\pi^-$ and $2\pi^+2\pi^-$

Sayipjamal Dulat[1,2,a] and Bing-Song Zou[3,1,b]

[1] Institute of High Energy Physics, CAS, P.O. Box 918(4), Beijing 100049, PRC
[2] Department of Physics, Xinjiang University, Urumqi 830046, PRC
[3] CCAST (World Laboratory), P.O. Box 8730, Beijing 100080, PRC

Received: 13 June 2005 / Revised version: 18 August 2005 /
Published online: 11 October 2005 – © Società Italiana di Fisica / Springer-Verlag 2005
Communicated by U.-G. Meißner

Abstract. With accumulation of high statistics data at BES and CLEO-c, many new interesting channels can get enough statistics for partial-wave analysis (PWA). Among them, $\psi \to \gamma p\bar{p}, \gamma\Lambda\bar{\Lambda}, \gamma\Sigma\bar{\Sigma}, \gamma\Xi\bar{\Xi}$ channels provide a good place for studying baryon-antibaryon interactions; the double radiative decays $\psi \to \gamma\gamma V$ with $V \equiv \rho, \omega, \phi$ have a potential to provide information on the flavor content of any meson resonances (R) with positive charge parity ($C = +$) and mass above 1 GeV through $\psi \to \gamma R \to \gamma\gamma V$; $\psi(2s) \to \gamma\chi_{c0,1,2}$ with $\chi_{c0,1,2} \to K\bar{K}\pi^+\pi^-$ and $2\pi^+2\pi^-$ decays are good processes to study χ_{cJ} charmonium decays. Using the covariant tensor formalism, here we provide theoretical PWA formulae for these channels.

PACS. 13.20.Gd Leptonic, semileptonic, and radiative decays of mesons: Decays of J/ψ, Υ, and other quarkonia – 13.25.Gv Hadronic decays of mesons: Decays of J/ψ, Υ, and other quarkonia – 13.66.Bc Hadron production in e^-e^+ interactions – 11.80.Et Partial-wave analysis

1 Introduction

Abundant J/ψ and ψ' events have been collected at the Beijing Electron Positron Collider (BEPC). More data will be collected at upgraded BEPC and CLEO-C. Many new interesting channels are now getting enough statistics for partial-wave analysis.

J/ψ and ψ' radiative decay to $B\bar{B}$ (baryon and antibaryon pair) is a good place to study baryon-antibaryon interactions and to look for possible resonant states of the $B\bar{B}$ system. Based on the 58 million J/ψ events accumulated by the BES2 detector at the BEPC, recently BES2 reported [1] that they observed a strong, narrow enhancement near the threshold in the invariant mass spectrum of $p\bar{p}$ (proton - antiproton) pairs from $J/\psi \to \gamma p\bar{p}$ radiative decays. The structure has attracted people's attention to study $p\bar{p}$ near-threshold interaction [2]. Future data on $\psi \to \gamma\Lambda\bar{\Lambda}, \gamma\Sigma\bar{\Sigma}, \gamma\Xi\bar{\Xi}$ channels will give a new opportunity to study hyperon-antihyperon interactions.

J/ψ and ψ' double radiative decays $\psi \to \gamma\gamma V(\rho, \omega, \phi)$ provide a favorable place to extract the $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$ structure of intermediate states [3]. The $J/\psi \to \gamma\gamma\rho$ and $\gamma\gamma\phi$ have been studied by Crystal Ball [4], DM2 [5], MARK-III [6] and BES-I [7]. An interesting structure at $\iota(1440)$ region in the γV invariant mass spectra is observed. But due to limited statistics, one cannot get reliable PWA results. With much higher statistics ψ data to be available soon at CLEO-C and BES-III, these ψ double radiative decay channels give a potential to provide information on the flavor content of any meson resonances (R) with positive charge parity ($C = +$) and mass above 1 GeV through $\psi \to \gamma R \to \gamma\gamma V$.

The $\psi(2s)$ radiative decays into $K^+K^-\pi^+\pi^-$ and $\pi^+\pi^-\pi^+\pi^-$ via χ_{cJ} intermediate states are good processes to study χ_{cJ} decays which may provide useful information on two-gluon hadronization dynamics and glueball decays.

In order to get more useful information about the resonance properties such as their J^{PC} quantum numbers, mass, width, production and decay rates, etc., partial-wave analyses (PWA) are necessary. PWA is an effective method for analysing the experimental data of hadron spectrum. There are two types of PWA: one is based on the covariant tensor (also named Rarita-Schwinger) formalism [8] and the other is based on the helicity formalism [9]. Reference [10] showed the connection between the covariant tensor formalism and the helicity one. Reference [11]

[a] e-mail: dulat98@yahoo.com; sdulat@xju.edu.cn
[b] e-mail: zoubs@mail.ihep.ac.cn

provided PWA formulae in the covariant tensor formalism for ψ decays to mesons, which have been used for a number of channels already published by BES [12] and are going to be used for more channels. A similar approach has been used in analyzing other reactions [13–15]. Reference [16] provided explicit formulae for the angular distribution of the photon in ψ radiative decays in the covariant tensor formalism, and also discussed helicity formalism of the angular distribution of the ψ radiative decays to two pseudoscalar mesons, and its relation to the covariant tensor formalism.

In this paper we extend the covariant tensor formalism [11] to give explicit PWA formulae for the new interesting channels mentioned above. The plan of this article is as follows: in sect. 2, we present the necessary tools for the calculation of the tensor amplitudes, within a covariant tensor formalism. This will allow us to derive covariant amplitudes for all possible processes. In sect. 3, we present covariant tensor formalism for ψ radiative decays to baryon antibaryon pairs. In sect. 4, we present covariant tensor formalism for ψ decays into $\gamma\gamma V(\rho,\omega,\phi)$. In sect. 5, we present covariant tensor formalism for the $\psi(2s)$ decays into $\gamma K^+K^-\pi^+\pi^-$ and $\gamma\pi^+\pi^-\pi^+\pi^-$, respectively. The conclusions are given in sect. 6. Since covariance is a useful property of any decay amplitude, all possible amplitudes are written in terms of covariant tensor form. All amplitudes include a complex coupling constant and Blatt-Weisskopf centrifugal barriers where necessary.

2 Prescriptions for the construction of covariant tensor amplitudes

In this section we present the necessary tools for the construction of covariant tensor amplitudes. Following the convention of ref. [11] for the ψ decays, the partial-wave amplitudes $U_i^{\mu\nu\alpha}$ in the covariant Rarita-Schwinger tensor formalism can be constructed by using pure orbital angular momentum covariant tensors $\tilde{t}^{(L_{bc})}_{\mu_1\cdots\mu_{L_{bc}}}$ and covariant spin wave functions $\phi_{\mu_1\cdots\mu_s}$ together with the metric tensor $g^{\mu\nu}$, the totally antisymmetric Levi-Civita tensor $\epsilon_{\mu\nu\lambda\sigma}$ and the four-momenta of participating particles; here the indices μ, ν, λ and α run from 1 to 4 over x, y, z and t. For a process $a\to bc$, if there exists a relative orbital angular momentum $\mathbf{L}_{bc}$ between the particle b and c, then the pure orbital angular momentum $\mathbf{L}_{bc}$ state can be represented by the covariant tensor wave function $\tilde{t}^{(L_{bc})}_{\mu_1\cdots\mu_{L_{bc}}}$ [9] which is built out of the relative momentum. Thus here we give only covariant tensors that correspond to the pure S-, P-, D-, and F-wave orbital angular momenta:

$$\tilde{t}^{(0)} = 1, \tag{1}$$

$$\tilde{t}^{(1)}_\mu = \tilde{g}_{\mu\nu}(p_a)r^\nu B_1(Q_{abc}) \equiv \tilde{r}_\mu B_1(Q_{abc}), \tag{2}$$

$$\tilde{t}^{(2)}_{\mu\nu} = [\tilde{r}_\mu\tilde{r}_\nu - \frac{1}{3}(\tilde{r}\cdot\tilde{r})\tilde{g}_{\mu\nu}(p_a)]B_2(Q_{abc}), \tag{3}$$

$$\tilde{t}^{(3)}_{\mu\nu\lambda} = \Big[\tilde{r}_\mu\tilde{r}_\nu\tilde{r}_\lambda - \frac{1}{5}(\tilde{r}\cdot\tilde{r})(\tilde{g}_{\mu\nu}(p_a)\tilde{r}_\lambda + \tilde{g}_{\nu\lambda}(p_a)\tilde{r}_\mu + \tilde{g}_{\lambda\mu}(p_a)\tilde{r}_\nu)\Big]B_3(Q_{abc}), \tag{4}$$

$$\ldots$$

$$p_a^\mu\tilde{t}^{(1)}_\mu = p_a^\mu\tilde{t}^{(2)}_{\mu\nu} = p_a^\mu t^{(3)}_{\mu\nu\lambda} = 0\,, \tag{5}$$

where $r = p_b - p_c$ is the relative four-momentum of the two decay products in the parent particle rest frame; $(\tilde{r}\cdot\tilde{r}) = -\mathbf{r}^2$ and

$$\tilde{g}_{\mu\nu}(p_a) = g_{\mu\nu} - \frac{p_{a\mu}p_{a\nu}}{p_a^2}\,. \tag{6}$$

Here the Minkowsky metric tensor has the form

$$g_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1).$$

$B_{L_{bc}}(Q_{abc})$ is a Blatt-Weisskopf barrier factor [9,17], where Q_{abc} is the magnitude of $\mathbf{p}_b$ or $\mathbf{p}_c$ in the rest system of a,

$$Q^2_{abc} = \frac{(s_a+s_b-s_c)^2}{4s_a} - s_b \tag{7}$$

with $s_a = E_a^2 - \mathbf{p}_a^2$.

The spin-1 and spin-2 particle wave functions $\phi_\mu(p_a,m_s)$ and $\phi_{\mu\nu}(p_a,m_s)$ with spin projection m_s satisfy the following conditions:

$$\begin{aligned} &p_a^\mu\phi_\mu(p_a,m_s) = 0, \quad \phi_\mu(p_a,m_s)\phi^{*\mu}(p_a,m_s') = -\delta_{m_sm_s'},\\ &\sum_{m_s}\phi_\mu(p_a,m_s)\phi^*_\nu(p_a,m_s) = -g_{\mu\nu} + \frac{p_{a\mu}p_{a\nu}}{p_a^2} \equiv -\tilde{g}_{\mu\nu}(p_a),\\ &p_a^\mu\phi_{\mu\nu}(p_a,m_s) = 0, \quad \phi_{\mu\nu} = \phi_{\nu\mu}, \quad g^{\mu\nu}\phi_{\mu\nu} = 0,\\ &\phi_{\mu\nu}(p_a,m_s)\phi^{*\mu\nu}(p_a,m_s') = \delta_{m_sm_s'}. \end{aligned} \tag{8}$$

Projection operators will be a useful general tool in constructing expressions. The spin-2 projection operator has the form [9,11]

$$P^{(2)}_{\mu\nu\mu'\nu'}(p_a) = \sum_{m_s}\phi_{\mu\nu}(p_a,m_s)\phi^*_{\mu'\nu'}(p_a,m_s) = \frac{1}{2}(\tilde{g}_{\mu\mu'}\tilde{g}_{\nu\nu'} + \tilde{g}_{\mu\nu'}\tilde{g}_{\nu\mu'}) - \frac{1}{3}\tilde{g}_{\mu\nu}\tilde{g}_{\mu'\nu'}. \tag{9}$$

Note that for a given decay process $a\to bc$, the total angular momentum should be conserved, which means

$$\mathbf{J}_a = \mathbf{S}_{bc} + \mathbf{L}_{bc}, \tag{10}$$

where

$$\mathbf{S}_{bc} = \mathbf{S}_b + \mathbf{S}_c. \tag{11}$$

In addition parity should also be conserved, which means

$$\eta_a = \eta_b\eta_c(-1)^{L_{bc}}, \tag{12}$$

where η_a, η_b, and η_c are the intrinsic parities of particles a, b, and c, respectively. From this relation, one knows whether L_{bc} should be even or odd. Then from eq. (10)

one can find out how many different (L_{bc}, S_{bc}) combinations there are, which determine the number of independent couplings. Also note that in the construction of the covariant tensor amplitude, if $S_{bc} + L_{bc} + J_a$ is an odd number, then $\epsilon_{\mu\nu\lambda\sigma} p_a^\sigma$ with p_a the momentum of the parent particle is needed; otherwise it is not needed. See, for example, eq. (28) below.

3 Covariant tensor formalism for ψ decay into $\gamma B\bar{B}$

The general form of the decay $\psi \to \gamma X \to \gamma p\bar{p}$ amplitude can be written as follows by using the polarization four-vectors of the initial and final states,

$$A^{(s)} = \psi_\mu(p, m_J) e_\nu^*(q, m_\gamma) \psi_{\alpha_s}(p_b, S_b; p_c, S_c) A^{\mu\nu\alpha_s}$$
$$= \psi_\mu(p, m_J) e_\nu^*(q, m_\gamma) \psi_{\alpha_s}(p_b, S_b; p_c, S_c) \sum_i \Lambda_i U_i^{\mu\nu\alpha_s}, \quad (13)$$

where $\psi_\mu(p, m_J)$ is the polarization four-vector of the ψ with spin projection of m_J; $e_\nu(q, m_\gamma)$ is the polarization four-vector of the photon with spin projections of m_γ; $\psi_{\alpha_s}(p_b, S_b; p_c, S_c)$ is the spin wave function of the proton and antiproton system with spin S_b and S_c, respectively, and the index s stands for the total spin of the $p\bar{p}$, see, for example, eqs. (18), (19); $U_i^{\mu\nu\alpha_s}$ is the i-th partial-wave amplitude with coupling strength determined by a complex parameter Λ_i. The spin-1 polarization vector $\psi_\mu(p, m_J)$ for ψ with four-momentum p_μ satisfies

$$\sum_{m_J=1}^{3} \psi_\mu(p, m_J) \psi_\nu^*(p, m_J) = -g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2} \equiv -\tilde{g}_{\mu\nu}(p), \quad (14)$$

with $p^\mu \psi_\mu = 0$. For ψ production from e^+e^- annihilation, the electrons are highly relativistic, with the result that $J_z = \pm 1$ for the ψ spin projection taking the beam direction as the z-axis. This limits m_J to 1 and 2, *i.e.* components along x and y. Then one has the following relation:

$$\sum_{m_J=1}^{2} \psi_\mu(p, m_J) \psi_{\mu'}^*(p, m_J) = \delta_{\mu\mu'}(\delta_{\mu 1} + \delta_{\mu 2}). \quad (15)$$

For the photon polarization four-vector, there is the usual Lorentz orthogonality condition. Namely, the polarization four-vector $e_\nu(q, m_\gamma)$ of the photon with momenta q satisfies

$$q^\nu e_\nu(q, m_\gamma) = 0, \quad (16)$$

which states that spin-1 wave function is orthogonal to its own momentum. The above relation is the same as for a massive vector meson. However, for the photon, there is an additional gauge invariance condition. Here we assume the Coulomb gauge in the ψ rest system, *i.e.*, $p^\nu e_\nu = 0$. Then we have [18]

$$\sum_{m_\gamma} e_\mu^*(q, m_\gamma) e_\nu(q, m_\gamma) =$$
$$-g_{\mu\nu} + \frac{q_\mu K_\nu + K_\mu q_\nu}{q \cdot K} - \frac{K \cdot K}{(q \cdot K)^2} q_\mu q_\nu \equiv -g_{\mu\nu}^{(\perp\perp)}, \quad (17)$$

with $K = p - q$ and $K^\nu e_\nu = 0$. We denote the four-momentum of the particle X by K, and $q \cdot K$ is a four-vector dot product. For $X \to p\bar{p}$, the total spin of $p\bar{p}$ system can be either 0 or 1. These two states can be represented by ψ and ψ_α [19],

$$\psi = \bar{u}(p_b, S_b) \gamma_5 v(p_c, S_c), \qquad \text{if } s = 0, \quad (18)$$
$$\psi_\alpha = \bar{u}(p_b, S_b) \left(\gamma_\alpha - \frac{r_\alpha}{m_X + 2m} \right) v(p_c, S_c), \quad \text{if } s = 1. \quad (19)$$

One can see that both ψ and ψ_α have no dependence on the direction of the momentum $\hat{\mathbf{p}}$, hence correspond to pure spin states with the total spin of 0 and 1, respectively. Here p_b, p_c, and S_b, S_c are momenta and spin of the proton antiproton pairs, respectively. m_X and m are the masses of X and p, $\bar{p}$, respectively; $u(p_b, S_b)$ and $v(p_c, S_c)$ are the standard Dirac spinors. If we sum over the polarization, we have the two projection operators

$$\sum_{S_b} u_\alpha(p_b, S_b) \bar{u}_\beta(p_b, S_b) = \left(\frac{\not{p}_b + m}{2m} \right)_{\alpha\beta},$$
$$\sum_{S_c} v_\alpha(p_c, S_c) \bar{v}_\beta(p_c, S_c) = \left(\frac{\not{p}_c - m}{2m} \right)_{\alpha\beta}. \quad (20)$$

To compute the differential cross-section, we need an expression for $|A|^2$. Note that the square modulus of the decay amplitude, which gives the decay probability of a certain configuration should be independent of any particular frame, that is, a Lorentz scalar. Thus by using eqs. (15) and (17), the differential cross-section for the radiative decay to 3-body final state is

$$\frac{d\sigma^{(s)}}{d\Phi_3} = \frac{1}{2} \sum_{S_b, S_c} \sum_{m_J=1}^{2} \sum_{m_\gamma=1}^{2} |\psi_\mu(p, m_J) e_\nu^*(q, m_\gamma)$$
$$\times \psi_{\alpha_s}(p_b, S_b; p_c, S_c) A^{\mu\nu\alpha_s}|^2$$
$$= -\frac{1}{2} \sum_{S_b, S_c} \sum_{\mu=1}^{2} A^{\mu\nu\alpha_s} g_{\nu\nu'}^{(\perp\perp)} A^{*\mu\nu'\alpha_s'} \psi_{\alpha_s}^* \psi_{\alpha_s'}$$
$$= -\frac{1}{2} \sum_{i,j} \Lambda_i \Lambda_j^* \sum_{\mu=1}^{2} U_i^{\mu\nu\alpha_s} g_{\nu\nu'}^{(\perp\perp)} U_j^{*\mu\nu'\alpha_s'} \sum_{S_b, S_c} \psi_{\alpha_s}^* \psi_{\alpha_s'}$$
$$\equiv \sum_{i,j} P_{ij} \cdot F_{ij}^{(s)}, \quad (21)$$

where

$$P_{ij} = P_{ji}^* = \Lambda_i \Lambda_j^*,$$
$$F_{ij}^{(s)} = F_{ji}^{*(s)} = -\frac{1}{2} \sum_{\mu=1}^{2} U_i^{\mu\nu\alpha_s} g_{\nu\nu'}^{(\perp\perp)} U_j^{*\mu\nu'\alpha_s'} \sum_{S_b, S_c} \psi_{\alpha_s}^* \psi_{\alpha_s'}. \quad (22)$$

$d\Phi_3$ is the standard Lorentz invariant 3-body phase space given by

$$d\Phi_3(p; q, p_b, p_c) = \delta^4(p - q - p_b - p_c) \frac{d^3\mathbf{q}}{(2\pi)^3 2E_\gamma}$$
$$\times \frac{m^2 d^3\mathbf{p}_b d^3\mathbf{p}_c}{(2\pi)^3 E_b (2\pi)^3 E_c}; \quad (23)$$

$$F_{ij}^{(0)} = F_{ji}^{*(0)} = -\frac{1}{2}\sum_{\mu=1}^{2} U_i^{\mu\nu} g_{\nu\nu'}^{(\perp\perp)} U_j^{*\mu\nu'} \sum_{S_b,S_c} \psi^*\psi$$
$$= \frac{1}{2}\sum_{\mu=1}^{2} U_i^{\mu\nu} g_{\nu\nu'}^{(\perp\perp)} U_j^{*\mu\nu'} \mathrm{Tr}\Big(\frac{\not p_b + m}{2m}\gamma_5 \frac{\not p_c - m}{2m}\gamma_5 \Big)$$
$$= -\frac{m_X^2}{4m^2}\sum_{\mu=1}^{2} U_i^{\mu\nu} g_{\nu\nu'}^{(\perp\perp)} U_j^{*\mu\nu'}. \quad (24)$$

The spin sums can be performed using the completeness relations from eq. (20):

$$F_{ij}^{(1)} = F_{ji}^{*(1)} = -\frac{1}{2}\sum_{\mu=1}^{2} U_i^{\mu\nu\alpha} g_{\nu\nu'}^{(\perp\perp)} U_j^{*\mu\nu'\alpha'} \sum_{S_b,S_c} \psi_\alpha^*\psi_{\alpha'}$$
$$= -\frac{1}{2}\sum_{\mu=1}^{2} U_i^{\mu\nu\alpha} g_{\nu\nu'}^{(\perp\perp)} U_j^{*\mu\nu'\alpha'}$$
$$\times\Big[\mathrm{Tr}\Big(\frac{\not p_b + m}{2m}\gamma_\alpha \frac{\not p_c - m}{2m}\gamma_{\alpha'} \Big)$$
$$-\frac{r_\alpha}{m_X+2m}\mathrm{Tr}\Big(\frac{\not p_b + m}{2m} \frac{\not p_c - m}{2m}\gamma_{\alpha'} \Big)$$
$$-\frac{r_{\alpha'}}{m_X+2m}\mathrm{Tr}\Big(\frac{\not p_b + m}{2m}\gamma_\alpha \frac{\not p_c - m}{2m} \Big)$$
$$+\frac{r_\alpha r_{\alpha'}}{(m_X+2m)^2}\mathrm{Tr}\Big(\frac{\not p_b + m}{2m} \frac{\not p_c - m}{2m} \Big)\Big]$$
$$= -\frac{1}{4m^2}\sum_{\mu=1}^{2} U_i^{\mu\nu\alpha} g_{\nu\nu'}^{(\perp\perp)} U_j^{*\mu\nu'\alpha'}\Big[p_{b\alpha}p_{b\alpha'} + p_{c\alpha}p_{c\alpha'}$$
$$+p_{b\alpha}p_{c\alpha'} + p_{b\alpha'}p_{c\alpha} - m_X^2 g_{\alpha\alpha'}\Big]. \quad (25)$$

3.1 Amplitudes for the radiative decay $\psi \to \gamma p\bar{p}$

We consider the decay of a ψ state in two steps: $\psi \to \gamma X$ with $X \to p\bar{p}$. The possible J^{PC} for X are $0^{++}, 0^{-+}, 1^{++}, 2^{++}, 2^{-+}$, etc. For $\psi \to \gamma X$, we choose two independent momenta p for ψ and q for the photon to be contracted with spin wave functions. We denote the four-momentum of X by K. The tensor describing the first and second steps will be denoted by $\tilde{T}^{(L)}_{\mu_1\cdots\mu_L}$ and $\tilde{t}^{(l)}_{\mu_1\cdots\mu_l}$, respectively.

For $\psi \to \gamma 0^{++} \to \gamma p\bar{p}$, there is one independent covariant tensor amplitude:

$$U^{\mu\nu\alpha} = g^{\mu\nu}\tilde{t}^{(1)\alpha}. \quad (26)$$

For $\psi \to \gamma 0^{-+} \to \gamma p\bar{p}$, there is one independent covariant tensor amplitude:

$$U^{\mu\nu} = \epsilon^{\mu\nu\lambda\sigma}p_\lambda q_\sigma B_1(Q_{\psi\gamma X}). \quad (27)$$

For $\psi \to \gamma 1^{++} \to \gamma p\bar{p}$, there are two independent covariant tensor amplitudes:

$$U_1^{\mu\nu\alpha} = \epsilon^{\mu\nu\lambda\sigma}p_\lambda \epsilon^{\alpha\beta\rho}{}_\sigma K_\rho \tilde{t}^{(1)}_\beta, \quad (28)$$
$$U_2^{\mu\nu\alpha} = \epsilon^{\nu\lambda\sigma\gamma}p_\lambda q^\mu q_\gamma \epsilon^{\alpha\beta\rho}{}_\sigma K_\rho \tilde{t}^{(1)}_\beta B_2(Q_{\psi\gamma X}). \quad (29)$$

For $\psi \to \gamma 1^{-+}$, the exotic 1^{-+} meson cannot decay into $p\bar{p}$.

For $\psi \to \gamma 2^{++} \to \gamma p\bar{p}$, there are six independent covariant tensor amplitudes:

$$U_1^{\mu\nu\alpha} = P^{(2)\mu\nu\alpha\beta}(K)\tilde{t}^{(1)}_\beta, \quad (30)$$
$$U_2^{\mu\nu\alpha} = P^{(2)\mu\nu\lambda\beta}(K)\tilde{t}^{(3)\alpha}_{\lambda\beta}, \quad (31)$$
$$U_3^{\mu\nu\alpha} = P^{(2)\nu\sigma\alpha\beta}q^\mu p_\sigma \tilde{t}^{(1)}_\beta B_2(Q_{\psi\gamma X}), \quad (32)$$
$$U_4^{\mu\nu\alpha} = P^{(2)\nu\sigma\lambda\beta}q^\mu p_\sigma \tilde{t}^{(3)\alpha}_{\lambda\beta} B_2(Q_{\psi\gamma X}), \quad (33)$$
$$U_5^{\mu\nu\alpha} = g^{\mu\nu}P^{(2)\sigma\rho\alpha\beta}p_\sigma p_\rho \tilde{t}^{(1)}_\beta B_2(Q_{\psi\gamma X}), \quad (34)$$
$$U_6^{\mu\nu\alpha} = g^{\mu\nu}P^{(2)\sigma\rho\lambda\beta}p_\sigma p_\rho \tilde{t}^{(3)\alpha}_{\lambda\beta} B_2(Q_{\psi\gamma X}). \quad (35)$$

where $\tilde{t}^{(1)}$ and $\tilde{t}^{(3)}$ correspond to the orbital angular momentum between the proton and antiproton l to be 1 and 3, respectively.

For $\psi \to \gamma 2^{-+} \to \gamma p\bar{p}$, the possible partial-wave amplitudes are the following:

$$U_1^{\mu\nu} = \epsilon^{\mu\nu\lambda\sigma}p_\lambda q^\gamma \tilde{t}^{(2)}_{\gamma\sigma} B_1(Q_{\psi\gamma X}), \quad (36)$$
$$U_2^{\mu\nu} = \epsilon^{\mu\nu\lambda\sigma}p_\lambda q_\sigma p_\gamma p_\delta \tilde{t}^{(2)\gamma\delta} B_3(Q_{\psi\gamma X}), \quad (37)$$
$$U_3^{\mu\nu} = \epsilon^{\nu\gamma\lambda\sigma}p_\lambda q_\sigma q^\mu p^\delta \tilde{t}^{(2)}_{\gamma\delta} B_3(Q_{\psi\gamma X}). \quad (38)$$

It is worth mentioning here that the above partial-wave amplitudes for the process $J/\psi \to \gamma p\bar{p}$ are applicable to the processes $J/\psi \to \gamma\Lambda\bar{\Lambda}, \gamma\Sigma\bar{\Sigma}$, and $\gamma\Xi\bar{\Xi}$ as well.

4 Covariant tensor formalism for ψ decay into $\gamma\gamma$V

By using the polarization four-vectors of the initial and final states, now we write the general form of the decay amplitude for the process

$$\psi \to \gamma R \to \gamma\gamma V(\rho,\phi,\omega) \quad (39)$$

as follows:

$$A = \psi_\mu(p,m_J)e^*_\nu(q,m_\gamma)\varepsilon^*_\alpha(k,m'_\gamma)A^{\mu\nu\alpha} =$$
$$\psi_\mu(p,m_J)e^*_\nu(q,m_\gamma)\varepsilon^*_\alpha(k,m'_\gamma)\sum_i \Lambda_i U_i^{\mu\nu\alpha}. \quad (40)$$

In the following $e_\nu(q,m_\gamma)$ denotes the polarization function of the photon in $\psi \to \gamma R$, and $\varepsilon_\alpha(k,m'_\gamma)$ denotes that of the photon in $R \to \gamma V$. The polarization four-vectors $\psi_\mu(p,m_J)$ and $e_\nu(q,m_\gamma)$ satisfy the conditions (14)-(17). And $\varepsilon_\alpha(k,m'_\gamma)$ satisfy

$$k^\alpha\varepsilon_\alpha(k,m'_\gamma) = 0, \quad (41)$$

$$\sum_{m'_\gamma}\varepsilon^*_\alpha(k,m'_\gamma)\varepsilon_\beta(k,m'_\gamma) =$$
$$-g_{\alpha\beta} + \frac{k_\alpha p_{V\beta} + p_{V\alpha}k_\beta}{k\cdot p_V} - \frac{p_V\cdot p_V}{(k\cdot p_V)^2}k_\alpha k_\beta \equiv -g^{(\perp)}_{\alpha\beta} \quad (42)$$

with $p_V = K - k$ and $p_V^\alpha \varepsilon_\alpha = 0$. We denote the four-momenta of the particles R and $V(\rho,\phi,\omega)$ by K and p_V, respectively. Then the differential cross-section for the radiative decay to an n-body final state is

$$\begin{aligned}\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_n} &= \frac{1}{2}\sum_{m_J=1}^{2}\sum_{m'_\gamma,m_\gamma=1}^{3}|\psi_\mu(p,m_J)e^*_\nu(q,m_\gamma)\varepsilon^*_\alpha(k,m'_\gamma)A^{\mu\nu\alpha}|^2\\ &= \frac{1}{2}\sum_{m_J=1}^{2}\psi_\mu(p,m_J)\psi^*_{\mu'}(p,m_J)g^{(\perp\perp)}_{\nu\nu'}g^{(\perp)}_{\alpha\alpha'}A^{\mu\nu\alpha}A^{*\mu'\nu'\alpha'}\\ &= \frac{1}{2}\sum_{\mu=1}^{2}A^{\mu\nu\alpha}g^{(\perp\perp)}_{\nu\nu'}g^{(\perp)}_{\alpha\alpha'}A^{*\mu\nu'\alpha'}\\ &= \frac{1}{2}\sum_{i,j}\Lambda_i\Lambda^*_j\sum_{\mu=1}^{2}U_i^{\mu\nu\alpha}g^{(\perp\perp)}_{\nu\nu'}g^{(\perp)}_{\alpha\alpha'}U_j^{*\mu\nu'\alpha'} \equiv \sum_{i,j}P_{ij}\cdot F_{ij},\end{aligned} \tag{43}$$

where

$$P_{ij} = P^*_{ji} = \Lambda_i\Lambda^*_j, \tag{44}$$

$$F_{ij} = F^*_{ji} = \frac{1}{2}\sum_{\mu=1}^{2}U_i^{\mu\nu\alpha}g^{(\perp\perp)}_{\nu\nu'}g^{(\perp)}_{\alpha\alpha'}U_j^{*\mu\nu'\alpha'}. \tag{45}$$

$\mathrm{d}\Phi_n$ is the standard element of n-body phase space given by

$$\mathrm{d}\Phi_n(p;p_1,\cdots p_n) = \delta^4(p - \sum_{i=1}^{n}p_i)\prod_{i=1}^{n}\frac{\mathrm{d}^3\mathbf{p}_i}{(2\pi)^3 2E_i}. \tag{46}$$

4.1 Amplitudes for the doubly radiative decay $\psi \to \gamma\gamma \mathbf{V}(\rho,\omega,\phi)$

This is a three step process: $\psi \to \gamma R$ with $R \to \gamma V(\rho,\omega,\phi)$ and $\rho \to \pi^+\pi^-$, $\omega \to \pi^0\pi^+\pi^-$, $\phi \to K^+K^-$, here we number π^0, π^+, π^- as 0, 1, 2. The intermediate resonance state R that may appear in the process with J^{PC} values are $0^{++}, 0^{-+}, 1^{++}, 1^{-+}, 2^{++}, 2^{-+}$, etc. Here J, P, C are the intrinsic spin, parity and C-parity of the R particle, respectively. For $\psi \to \gamma R$, we denote the spin-orbital angular momenta between the photon and ψ by S and L, respectively. The tensor describing the first and the second steps will be denoted by $\tilde{T}^{(L)}_{\mu_1\cdots\mu_L}$ and $\tilde{t}^{(L_1)}_{\mu_1\cdots\mu_{L_1}}$, respectively. The vector describing the third step will be denoted by V_μ, where $V(\rho,\phi)_\mu = p_{1\mu} - p_{2\mu}$, here we use the fact that π^+ and π^- (or K^+ and K^-) have equal masses; and

$$\begin{aligned}V(\omega)_\mu &= \epsilon^{\mu}{}_{\nu\lambda\sigma}p_1^\nu p_2^\lambda p_0^\sigma[B_1(Q_{\omega\rho 0})f^{(\rho)}_{(12)}B_1(Q_{\rho 12})\\ &+B_1(Q_{\omega\rho 2})f^{(\rho)}_{(01)}B_1(Q_{\rho 10}) + B_1(Q_{\omega\rho 1})f^{(\rho)}_{(02)}B_1(Q_{\rho 20})].\end{aligned}$$

Now we write the decay amplitude of the ψ into two photons and a vector in a general and compact form using the covariant tensor formalism. There is one independent covariant tensor amplitude for $\psi \to \gamma 0^{++} \to \gamma\gamma V(\rho,\omega,\phi)$

$$U^{\mu\nu\alpha} = g^{\mu\nu}V^\alpha f^{(R)}f^{(V)}, \tag{47}$$

where $f^{(V)}$ either $f^{(\rho,\phi)}_{(12)}$ or $f^{(\omega)}_{(012)}$.

There is also one independent covariant tensor amplitude for $\psi \to \gamma 0^{-+} \to \gamma\gamma V(\rho,\omega,\phi)$

$$U^{\mu\nu\alpha} = \epsilon^{\mu\nu\lambda\sigma}p_\lambda\tilde{T}^{(1)}_\sigma\epsilon^{\alpha\beta\rho\delta}K_\rho t^{(1)}_{1\beta}V_\delta f^{(R)}f^{(V)}. \tag{48}$$

For the production reaction $\psi \to \gamma 1^{++}$ there are two independent covariant tensor amplitudes; there are also two amplitudes for the decay reaction $1^{++} \to \gamma V(\rho,\omega,\phi)$, all in all we have four amplitudes:

$$U_1^{\mu\nu\alpha} = \epsilon^{\mu\nu\lambda\sigma}p_\lambda\epsilon^{\alpha\beta\rho}{}_\sigma K_\rho V_\beta f^{(R)}f^{(V)}, \tag{49}$$

$$U_2^{\mu\nu\alpha} = \epsilon^{\mu\nu\lambda\sigma}p_\lambda\tilde{T}^{(2)}_{\sigma\gamma}\epsilon^{\alpha\beta\rho\delta}K_\rho\tilde{t}^{(2)\gamma}_\delta V_\beta f^{(R)}f^{(V)}, \tag{50}$$

$$U_3^{\mu\nu\alpha} = \epsilon^{\mu\nu\lambda\sigma}p_\lambda\epsilon^{\alpha\beta\rho\delta}K_\rho\tilde{t}^{(2)}_{\sigma\delta}V_\beta f^{(R)}f^{(V)}, \tag{51}$$

$$U_4^{\mu\nu\alpha} = \epsilon^{\mu\nu\lambda\sigma}p_\lambda\tilde{T}^{(2)}_{\sigma\delta}\epsilon^{\alpha\beta\rho\delta}K_\rho V_\beta f^{(R)}f^{(V)}. \tag{52}$$

For the production reaction $\psi \to \gamma 1^{-+}$ there are two independent covariant tensor amplitudes; there are also two amplitudes for the decay reaction $1^{-+} \to \gamma V(\rho,\omega,\phi)$, all in all we have four amplitudes:

$$U_1^{\mu\nu\alpha} = g^{\mu\nu}\tilde{T}^{(1)}_\beta\tilde{t}^{(1)\beta}V^\alpha f^{(R)}f^{(V)}, \tag{53}$$

$$U_2^{\mu\nu\alpha} = \tilde{T}^{(1)\mu}\tilde{t}^{(1)\nu}V^\alpha f^{(R)}f^{(V)}, \tag{54}$$

$$U_3^{\mu\nu\alpha} = g^{\mu\nu}\tilde{T}^{(1)\alpha}\tilde{t}^{(1)\beta}V_\beta f^{(R)}f^{(V)}, \tag{55}$$

$$U_4^{\mu\nu\alpha} = \tilde{T}^{(1)\mu}g^{\nu\alpha}\tilde{t}^{(1)\beta}V_\beta f^{(R)}f^{(V)}. \tag{56}$$

For the production reaction $\psi \to \gamma 2^{++}$ there are three independent covariant tensor amplitudes; there are also three amplitudes for the decay reaction $2^{++} \to \gamma V(\rho,\omega,\phi)$, all in all we have nine amplitudes:

$$U_1^{\mu\nu\alpha} = P^{(2)\mu\nu\alpha\beta}(K)V_\beta f^{(R)}f^{(V)}, \tag{57}$$

$$U_2^{\mu\nu\alpha} = g^{\mu\nu}P^{(2)\lambda\sigma\rho\delta}(K)\tilde{T}^{(2)}_{\lambda\sigma}\tilde{t}^{(2)}_{\rho\delta}V^\alpha f^{(R)}f^{(V)}, \tag{58}$$

$$U_3^{\mu\nu\alpha} = P^{(2)\nu\sigma\alpha\lambda}(K)\tilde{T}^{(2)\mu}_\sigma\tilde{t}^{(2)}_{\lambda\beta}V^\beta f^{(R)}f^{(V)}, \tag{59}$$

$$U_4^{\mu\nu\alpha} = P^{(2)\mu\nu\lambda\sigma}(K)\tilde{t}^{(2)}_{\lambda\sigma}V^\alpha f^{(R)}f^{(V)}, \tag{60}$$

$$U_5^{\mu\nu\alpha} = P^{(2)\mu\nu\alpha\lambda}(K)\tilde{t}^{(2)}_{\beta\lambda}V^\beta f^{(R)}f^{(V)}, \tag{61}$$

$$U_6^{\mu\nu\alpha} = g^{\mu\nu}P^{(2)\lambda\sigma\alpha\beta}(K)\tilde{T}^{(2)}_{\lambda\sigma}V_\beta f^{(R)}f^{(V)}, \tag{62}$$

$$U_7^{\mu\nu\alpha} = g^{\mu\nu}P^{(2)\lambda\sigma\alpha\delta}(K)\tilde{T}^{(2)}_{\lambda\sigma}\tilde{t}^{(2)}_{\beta\delta}V^\beta f^{(R)}f^{(V)}, \tag{63}$$

$$U_8^{\mu\nu\alpha} = P^{(2)\nu\lambda\alpha\beta}(K)\tilde{T}^{(2)\mu}_\lambda V_\beta f^{(R)}f^{(V)}, \tag{64}$$

$$U_9^{\mu\nu\alpha} = P^{(2)\nu\delta\lambda\sigma}(K)\tilde{T}^{(2)\mu}_\delta\tilde{t}^{(2)}_{\lambda\sigma}V^\alpha f^{(R)}f^{(V)}. \tag{65}$$

For the production reaction $\psi \to \gamma 2^{-+}$ there are three independent covariant tensor amplitudes; there are also three amplitudes for the decay reaction $2^{-+} \to \gamma V(\rho,\omega,\phi)$, all in all we have nine amplitudes:

$$U_1^{\mu\nu\alpha} = \epsilon^{\mu\nu\lambda\sigma} p_\sigma \tilde{T}^{(1)\gamma} \epsilon^{\alpha\beta\rho\xi} K_\xi \tilde{t}^{(1)\delta} \times P^{(2)}_{\lambda\gamma\rho\delta}(K) V_\beta f^{(R)} f^{(V)}, \quad (66)$$

$$U_2^{\mu\nu\alpha} = \epsilon^{\mu\nu\lambda\sigma} p_\sigma \tilde{T}^{(3)}_{\lambda\gamma\delta} \epsilon^{\alpha\beta\rho\xi} K_\xi \tilde{t}^{(3)}_{\rho\gamma'\delta'} \times P^{(2)\gamma\delta\gamma'\delta'}(K) V_\beta f^{(R)} f^{(V)}, \quad (67)$$

$$U_3^{\mu\nu\alpha} = \epsilon^{\nu\lambda\sigma\gamma} p_\gamma \tilde{T}^{(3)\mu\lambda'}_{\sigma} \epsilon^{\beta\rho\delta\xi} K_\xi \tilde{t}^{(3)\alpha\rho'}_{\delta} \times P^{(2)}_{\lambda\lambda'\rho\rho'}(K) V_\beta f^{(R)} f^{(V)}, \quad (68)$$

$$U_4^{\mu\nu\alpha} = \epsilon^{\mu\nu\lambda\sigma} p_\sigma \tilde{T}^{(1)\gamma} \epsilon^{\alpha\beta\rho\xi} K_\xi \tilde{t}^{(3)\delta\zeta}_{\rho} \times P^{(2)}_{\lambda\gamma\delta\zeta}(K) V_\beta f^{(R)} f^{(V)}, \quad (69)$$

$$U_5^{\mu\nu\alpha} = \epsilon^{\mu\nu\lambda\sigma} p_\sigma \tilde{T}^{(1)\gamma} \epsilon^{\beta\rho\delta\xi} K_\xi \times P^{(2)}_{\lambda\gamma\rho\zeta}(K) \tilde{t}^{(3)\alpha\zeta}_{\delta} V_\beta f^{(R)} f^{(V)}, \quad (70)$$

$$U_6^{\mu\nu\alpha} = \epsilon^{\mu\nu\lambda\sigma} p_\sigma \tilde{T}^{(3)\gamma\delta}_{\lambda} \epsilon^{\alpha\beta\rho\xi} K_\xi \tilde{t}^{(1)\zeta} \times P^{(2)}_{\gamma\delta\rho\zeta}(K) V^\beta f^{(R)} f^{(V)}, \quad (71)$$

$$U_7^{\mu\nu\alpha} = \epsilon^{\mu\nu\lambda\sigma} p_\sigma \tilde{T}^{(3)\gamma\delta}_{\lambda} \epsilon^{\beta\tau\rho\xi} K_\xi \tilde{t}^{(3)\alpha\delta'}_{\rho} \times P^{(2)}_{\gamma\delta\tau\delta'}(K) V_\beta f^{(R)} f^{(V)}, \quad (72)$$

$$U_8^{\mu\nu\alpha} = \epsilon^{\nu\lambda\sigma\gamma} p_\gamma \tilde{T}^{(3)\mu\zeta}_{\sigma} \epsilon^{\alpha\beta\rho\xi} K_\xi \tilde{t}^{(1)\delta} \times P^{(2)}_{\lambda\zeta\rho\delta}(K) V_\beta f^{(R)} f^{(V)}, \quad (73)$$

$$U_9^{\mu\nu\alpha} = \epsilon^{\nu\lambda\sigma\gamma} p_\gamma \tilde{T}^{(3)\mu\delta}_{\sigma} \epsilon^{\alpha\beta\rho\xi} K_\xi \tilde{t}^{(3)\lambda'\delta'}_{\rho} \times P^{(2)}_{\lambda\delta\lambda'\delta'}(K) V_\beta f^{(R)} f^{(V)}. \quad (74)$$

5 Formalism for $\psi(2s) \to \gamma\chi_{cJ}$ with $\chi_{cJ} \to K\bar{K}\pi^+\pi^-$ and $2\pi^+2\pi^-$

By following ref. [11] we denote the $\psi(2s)$ polarization four-vector by $\psi_\mu(p, m_J)$ and the photon polarization vector by $e_\nu(q, m_\gamma)$. Then the general form for the decay amplitude is

$$A = \psi_\mu(p,m_J) e^*_\nu(q,m_\gamma) A^{\mu\nu} = \psi_\mu(p,m_J) e^*_\nu(q,m_\gamma) \sum_i \Lambda_i U_i^{\mu\nu}. \quad (75)$$

The radiative decay cross-section is given in

$$\begin{aligned}\frac{d\sigma}{d\Phi_n} &= \frac{1}{2}\sum_{m_J=1}^{2}\sum_{m_\gamma=1}^{2} \psi_\mu(p,m_J) e^*_\nu(q,m_\gamma) \times A^{\mu\nu} \psi^*_{\mu'}(p,m_J) e_{\nu'}(q,m_\gamma) A^{*\mu'\nu'} \\ &= -\frac{1}{2}\sum_{m_J=1}^{2} \psi_\mu(p,m_J)\psi^*_{\mu'}(p,m_J) g^{(\perp\perp)}_{\nu\nu'} A^{\mu\nu} A^{*\mu'\nu'} \\ &= -\frac{1}{2}\sum_{\mu=1}^{2} A_{\mu\nu} g^{(\perp\perp)}_{\nu\nu'} A^{*\mu\nu'} \\ &= -\frac{1}{2}\sum_{i,j} \Lambda_i \Lambda_j^* \sum_{\mu=1}^{2} U_i^{\mu\nu} g^{(\perp\perp)}_{\nu\nu'} U_j^{*\mu\nu'} \equiv \sum_{i,j} P_{ij}\cdot F_{ij}, \end{aligned} \quad (76)$$

where $g^{(\perp\perp)}_{\nu\nu'}$ is given in (17) and

$$P_{ij} = P^*_{ji} = \Lambda_i \Lambda_j^*, \quad (77)$$

$$F_{ij} = F^*_{ji} = -\frac{1}{2}\sum_{\mu=1}^{2} U_i^{\mu\nu} g^{(\perp\perp)}_{\nu\nu'} U_j^{*\mu\nu'}. \quad (78)$$

Note that due to the special properties (massless and gauge invariance) of the photon, the number of independent partial-wave amplitudes for a $\psi(2s)$ radiative decay is smaller than for the corresponding decay to a massive vector meson [11]. We come now to specific examples of reactions.

5.1 $\psi \to \gamma\chi_{c0} \to \gamma K^+K^-\pi^+\pi^-$

We construct the covariant amplitudes $U^i_{\mu\nu}$ for this channel. Here we number K^+, K^-, π^+, π^- as 1, 2, 3, 4.

$$\langle K_0^*\bar{K}_0^*|1\rangle = g_{\mu\nu} f^{(K_0^*)}_{(14)} f^{(\bar{K}_0^*)}_{(23)}, \quad (79)$$

$$\langle K_0^*\bar{K}_2^*|1\rangle = g_{\mu\nu} \tilde{T}^{(2)\alpha\beta}_{[K_0\bar{K}_2]} \tilde{t}^{(2)}_{(23)\alpha\beta} f^{(K_0^*)}_{(14)} f^{(\bar{K}_2^*)}_{(23)} + \{1\leftrightarrow 2, 3\leftrightarrow 4\}, \quad (80)$$

$$\langle K_2^*\bar{K}_2^*|1\rangle = g_{\mu\nu} \tilde{t}^{(2)\alpha\beta}_{(14)} \tilde{t}^{(2)}_{(23)\alpha\beta} f^{(K_2^*)}_{(14)} f^{(\bar{K}_2^*)}_{(23)}, \quad (81)$$

$$\langle K_2^*\bar{K}_2^*|2\rangle = g_{\mu\nu} \tilde{T}^{(2)\alpha\beta}_{[K_2\bar{K}_2]} \tilde{t}^{(2)\gamma}_{(14)\alpha} \tilde{t}^{(2)}_{(23)\beta\gamma} f^{(K_2^*)}_{(14)} f^{(\bar{K}_2^*)}_{(23)}, \quad (82)$$

$$\langle K^*\bar{K}^*|1\rangle = g_{\mu\nu} \tilde{t}^{(1)\alpha}_{(14)} \tilde{t}^{(1)}_{(23)\alpha} f^{(K^*)}_{(14)} f^{(\bar{K}^*)}_{(23)}, \quad (83)$$

$$\langle K^*\bar{K}^*|2\rangle = g_{\mu\nu} \tilde{T}^{(2)\alpha\beta}_{[K^*\bar{K}^*]} \tilde{t}^{(1)}_{(14)\alpha} \tilde{t}^{(1)}_{(23)\beta} f^{(K^*)}_{(14)} f^{(\bar{K}^*)}_{(23)}, \quad (84)$$

$$\langle K'K|K\rho\rangle = g_{\mu\nu} \tilde{T}^{(1)\alpha}_{[K\rho]} \tilde{t}^{(1)}_{(34)\alpha} f^{(K')}_{(134)} f^{(\rho)}_{(34)} + \{1\leftrightarrow 2, 3\leftrightarrow 4\}, \quad (85)$$

$$\langle K'K|K^*\pi\rangle = g_{\mu\nu} \tilde{T}^{(1)\alpha}_{[K^*3]} \tilde{t}^{(1)}_{(14)\alpha} f^{(K')}_{(134)} f^{(K^*)}_{(14)} + \{1\leftrightarrow 2, 3\leftrightarrow 4\}, \quad (86)$$

$$\langle K'K|K_0^*\pi\rangle = g_{\mu\nu} f^{(K')}_{(134)} f^{(K_0^*)}_{(14)} + \{1\leftrightarrow 2, 3\leftrightarrow 4\}, \quad (87)$$

$$\langle K_1^*K|K\rho\rangle = g_{\mu\nu} \tilde{g}_{\alpha\beta}(p_{K_1^*}) \tilde{T}^{(1)\alpha}_{[K_1^*\bar{K}]} \tilde{t}^{(1)\beta}_{(34)} f^{(K_1^*)}_{(134)} f^{(\rho)}_{(34)} + \{1\leftrightarrow 2, 3\leftrightarrow 4\}, \quad (88)$$

$$\langle K_1^*K|K^*\pi\rangle_1 = g_{\mu\nu} \tilde{g}_{\alpha\beta}(p_{K_1^*}) \tilde{T}^{(1)\alpha}_{[K_1^*\bar{K}]} \tilde{t}^{(1)\beta}_{(14)} f^{(K_1^*)}_{(134)} f^{(K^*)}_{(14)} + \{1\leftrightarrow 2, 3\leftrightarrow 4\}, \quad (89)$$

$$\langle K_1^*K|K^*\pi\rangle_2 = g_{\mu\nu} \tilde{g}_{\alpha\beta}(p_{K_1^*}) \tilde{T}^{(1)\alpha}_{[K_1^*\bar{K}]} \tilde{t}^{(2)\beta\sigma}_{(K^*\pi)} \tilde{t}^{(1)}_{(14)\sigma} \times f^{(K_1^*)}_{(134)} f^{(K^*)}_{(14)} + \{1\leftrightarrow 2, 3\leftrightarrow 4\}, \quad (90)$$

$$\langle K_1^*K|K_0^*\pi\rangle = g_{\mu\nu} \tilde{g}_{\alpha\beta}(p_{K_1^*}) \tilde{T}^{(1)\alpha}_{[K_1^*\bar{K}]} \tilde{t}^{(1)\beta}_{(K_0^*\pi)} f^{(K_1^*)}_{(134)} f^{(K_0^*)}_{(14)} + \{1\leftrightarrow 2, 3\leftrightarrow 4\}, \quad (91)$$

$$\langle K_2K|K^*\pi\rangle = g_{\mu\nu} P^{(2)}_{\alpha\beta\lambda\sigma}(p_{K_2}) \tilde{T}^{(2)\alpha\beta}_{[K_2\bar{K}]} \tilde{t}^{(1)\sigma}_{(K^*\pi)} \tilde{t}^{(1)\lambda}_{(14)} \times f^{(K_1^*)}_{(134)} f^{(K^*)}_{(14)} + \{1\leftrightarrow 2, 3\leftrightarrow 4\}, \quad (92)$$

$$\langle f_0 f_0'|1\rangle = g_{\mu\nu} f^{(f_0)}_{(34)} f^{(f_0')}_{(12)}, \quad (93)$$

$$\langle f_0 f_2|1\rangle = g_{\mu\nu} \tilde{T}^{(2)\alpha\beta}_{[f_0 f_2]} \tilde{t}^{(2)}_{(34)\alpha\beta} f^{(f_0)}_{(34)} f^{(f_2)}_{(12)}, \quad (94)$$

$$\langle f_2 f_2'|1\rangle = g_{\mu\nu} \tilde{t}^{(2)\alpha\beta}_{(12)} \tilde{t}^{(2)}_{(34)\alpha\beta} f^{(f_2)}_{(12)} f^{(f_2')}_{(34)}, \quad (95)$$

$$\langle f_2 f_2'|2\rangle = g_{\mu\nu} \tilde{T}^{(2)\alpha\beta}_{[f_2 f_2']} \tilde{t}^{(2)\gamma}_{(12)\alpha} \tilde{t}^{(2)}_{(34)\beta\gamma} f^{(f_2)}_{(12)} f^{(f_2')}_{(34)}. \quad (96)$$

5.2 $\psi \to \gamma\chi_{c1} \to \gamma K^+K^-\pi^+\pi^-$

In this subsection we construct the amplitudes $U^i_{\mu\nu}$ for the process $\psi \to \gamma\chi_{c1} \to \gamma K^+K^-\pi^+\pi^-$. The most likely intermediate states are: $K_0^*\bar{K}^*$, $K_0^*\bar{K}_2^*$, $K_2^*\bar{K}^*$, $K_2^*\bar{K}_2^*$, $K^*\bar{K}^*$, K_1^*K, K_2^*K with K_0^*, K_2^*, $K^* \to K\pi$, $K_1^* \to \rho K$, $K^*\pi$, $K_0^*\pi$, and f_0f_2, f_2f_2' with $f_0 \to \pi^+\pi^-$, $f_0' \to K^+K^-$, $f_2 \to K^+K^-$ and $f_2' \to \pi^+\pi^-$.

$$\begin{aligned}\langle K_0^*\bar{K}^*|1\rangle = {}& \varepsilon_{\mu\nu\alpha\beta}p_\psi^\beta\varepsilon_{\lambda\gamma\delta\xi}p^\xi_{\chi_{c1}}g^{\alpha\delta}[\tilde{t}^{(1)\gamma}_{[K_0^*\bar{K}^*]}\tilde{t}^{(1)\lambda}_{(23)}\\ &\times f^{(K_0^*)}_{(14)}f^{(\bar{K}^*)}_{(23)} + \{1\leftrightarrow 2, 3\leftrightarrow 4\}],\end{aligned}\tag{97}$$

$$\begin{aligned}\langle K_0^*\bar{K}^*|2\rangle = {}& \varepsilon_{\rho\nu\alpha\beta}p_\psi^\beta\varepsilon_{\lambda\gamma\delta\xi}p^\xi_{\chi_{c1}}q^\rho q_\mu g^{\alpha\delta}[\tilde{t}^{(1)\gamma}_{[K_0^*\bar{K}^*]}\tilde{t}^{(1)\lambda}_{(23)}\\ &\times f^{(K_0^*)}_{(14)}f^{(\bar{K}^*)}_{(23)} + \{1\leftrightarrow 2, 3\leftrightarrow 4\}],\end{aligned}\tag{98}$$

$$\begin{aligned}\langle K_0^*\bar{K}_2^*|1\rangle = {}& \varepsilon_{\mu\nu\alpha\beta}p_\psi^\beta\varepsilon_{\lambda\gamma\delta\xi}p^\xi_{\chi_{c1}}g^{\alpha\delta}[\tilde{T}^{(2)\gamma}_{[K_0^*\bar{K}_2^*]\sigma}\tilde{t}^{(2)\lambda\sigma}_{(23)}\\ &\times f^{(K_0^*)}_{(14)}f^{(\bar{K}_2^*)}_{(23)} + \{1\leftrightarrow 2, 3\leftrightarrow 4\}],\end{aligned}\tag{99}$$

$$\begin{aligned}\langle K_0^*\bar{K}_2^*|2\rangle = {}& \varepsilon_{\rho\nu\alpha\beta}p_\psi^\beta q^\rho q_\mu\varepsilon_{\lambda\gamma\delta\xi}p^\xi_{\chi_{c1}}g^{\alpha\delta}[\tilde{T}^{(2)\gamma}_{[K_0^*\bar{K}_2^*]\sigma}\tilde{t}^{(2)\lambda\sigma}_{(23)}\\ &\times f^{(K_0^*)}_{(14)}f^{(\bar{K}_2^*)}_{(23)} + \{1\leftrightarrow 2, 3\leftrightarrow 4\}],\end{aligned}\tag{100}$$

$$\begin{aligned}\langle K_2^*\bar{K}^*|1\rangle = {}& \varepsilon_{\mu\nu\alpha\beta}p_\psi^\beta\varepsilon_{\lambda\gamma\delta\xi}p^\xi_{\chi_{c1}}g^{\alpha\delta}[\tilde{t}^{(1)\gamma}_{[K_2^*\bar{K}^*]}\tilde{t}^{(2)\lambda\sigma}_{(14)}\tilde{t}^{(1)}_{(23)\sigma}\\ &\times f^{(K_2^*)}_{(14)}f^{(\bar{K}^*)}_{(23)} + \{1\leftrightarrow 2, 3\leftrightarrow 4\}],\end{aligned}\tag{101}$$

$$\begin{aligned}\langle K_2^*\bar{K}^*|2\rangle = {}& \varepsilon_{\mu\nu\alpha\beta}p_\psi^\beta\varepsilon_{\lambda\gamma\delta\xi}p^\xi_{\chi_{c1}}g^{\alpha\delta}[\tilde{t}^{(1)}_{[K_2^*\bar{K}^*]\sigma}\tilde{t}^{(2)\lambda\sigma}_{(14)}\tilde{t}^{(1)\gamma}_{(23)}\\ &\times f^{(K_2^*)}_{(14)}f^{(\bar{K}^*)}_{(23)} + \{1\leftrightarrow 2, 3\leftrightarrow 4\}],\end{aligned}\tag{102}$$

$$\begin{aligned}\langle K_2^*\bar{K}^*|3\rangle = {}& \varepsilon_{\rho\nu\alpha\beta}p_\psi^\beta q^\rho q_\mu\varepsilon_{\lambda\gamma\delta\xi}p^\xi_{\chi_{c1}}g^{\alpha\delta}[\tilde{t}^{(1)\gamma}_{[K_2^*\bar{K}^*]}\tilde{t}^{(2)\lambda\sigma}_{(14)}\\ &\times\tilde{t}^{(1)}_{(23)\sigma}f^{(K_2^*)}_{(14)}f^{(\bar{K}^*)}_{(23)} + \{1\leftrightarrow 2, 3\leftrightarrow 4\}],\end{aligned}\tag{103}$$

$$\begin{aligned}\langle K_2^*\bar{K}^*|4\rangle = {}& \varepsilon_{\rho\nu\alpha\beta}p_\psi^\beta q^\rho q_\mu\varepsilon_{\lambda\gamma\delta\xi}p^\xi_{\chi_{c1}}g^{\alpha\delta}[\tilde{t}^{(1)}_{[K_2^*\bar{K}^*]\sigma}\tilde{t}^{(2)\lambda\sigma}_{(14)}\\ &\times\tilde{t}^{(1)\gamma}_{(23)}f^{(K_2^*)}_{(14)}f^{(\bar{K}^*)}_{(23)} + \{1\leftrightarrow 2, 3\leftrightarrow 4\}],\end{aligned}\tag{104}$$

$$\begin{aligned}\langle K_2^*\bar{K}_2^*|1\rangle = {}& \varepsilon_{\mu\nu\alpha\beta}p_\psi^\beta\varepsilon_{\lambda\gamma\delta\xi}p^\xi_{\chi_{c1}}g^{\alpha\delta}\tilde{t}^{(2)\lambda\sigma}_{(14)}\tilde{t}^{(2)\gamma}_{(23)\sigma}\\ &\times f^{(K_2^*)}_{(14)}f^{(\bar{K}_2^*)}_{(23)},\end{aligned}\tag{105}$$

$$\begin{aligned}\langle K_2^*\bar{K}_2^*|2\rangle = {}& \varepsilon_{\rho\nu\alpha\beta}p_\psi^\beta q^\rho q_\mu\varepsilon_{\lambda\gamma\delta\xi}p^\xi_{\chi_{c1}}g^{\alpha\delta}\tilde{t}^{(2)\lambda\sigma}_{(14)}\tilde{t}^{(2)\gamma}_{(23)\sigma}\\ &\times f^{(K_2^*)}_{(14)}f^{(\bar{K}_2^*)}_{(23)},\end{aligned}\tag{106}$$

$$\begin{aligned}\langle K^*\bar{K}^*|1\rangle = {}& \varepsilon_{\mu\nu\alpha\beta}p_\psi^\beta\varepsilon_{\lambda\sigma\gamma\delta}p^\delta_{\chi_{c1}}g^{\alpha\gamma}\tilde{t}^{(1)\lambda}_{(14)}\tilde{t}^{(1)\sigma}_{(23)}\\ &\times f^{(K^*)}_{(14)}f^{(\bar{K}^*)}_{(23)},\end{aligned}\tag{107}$$

$$\begin{aligned}\langle K^*\bar{K}^*|2\rangle = {}& \varepsilon_{\rho\nu\alpha\beta}p_\psi^\beta q^\rho q_\mu\varepsilon_{\lambda\sigma\gamma\delta}p^\delta_{\chi_{c1}}g^{\alpha\gamma}\tilde{t}^{(1)\lambda}_{(14)}\tilde{t}^{(1)\sigma}_{(23)}\\ &\times f^{(K^*)}_{(14)}f^{(\bar{K}^*)}_{(23)},\end{aligned}\tag{108}$$

$$\begin{aligned}\langle K_1^*K|K\rho\rangle_1 = {}& \varepsilon_{\mu\nu\alpha\beta}p_\psi^\beta\varepsilon_{\lambda\sigma\gamma\delta}p^\delta_{\chi_{c1}}g^{\alpha\gamma}\tilde{T}^{(1)\sigma}_{[K_1^*K]}\tilde{g}^{\lambda\xi}(p_{K_1^*})\\ &\times[\tilde{t}^{(1)}_{(34)\xi}f^{(K_1^*)}_{(134)}f^{(\rho)}_{(34)} + \{1\leftrightarrow 2, 3\leftrightarrow 4\}],\end{aligned}\tag{109}$$

$$\begin{aligned}\langle K_1^*K|K\rho\rangle_2 = {}& \varepsilon_{\eta\nu\alpha\beta}p_\psi^\beta q^\eta q_\mu\varepsilon_{\lambda\sigma\gamma\delta}p^\delta_{\chi_{c1}}g^{\alpha\gamma}\tilde{T}^{(1)\sigma}_{[K_1^*K]}\tilde{g}^{\lambda\xi}(p_{K_1^*})\\ &\times[\tilde{t}^{(1)}_{(34)\xi}f^{(K_1^*)}_{(134)}f^{(\rho)}_{(34)} + \{1\leftrightarrow 2, 3\leftrightarrow 4\}],\end{aligned}\tag{110}$$

$$\begin{aligned}\langle K_1^*K|K^*\pi\rangle_1 = {}& \varepsilon_{\mu\nu\alpha\beta}p_\psi^\beta\varepsilon_{\lambda\sigma\gamma\delta}p^\delta_{\chi_{c1}}g^{\alpha\gamma}\tilde{T}^{(1)\sigma}_{[K_1^*K]}\tilde{g}^{\lambda\xi}(p_{K_1^*})\\ &\times[\tilde{t}^{(1)}_{(14)\xi}f^{(K_1^*)}_{(134)}f^{(K^*)}_{(14)} + \{1\leftrightarrow 2, 3\leftrightarrow 4\}],\end{aligned}\tag{111}$$

$$\begin{aligned}\langle K_1^*K|K^*\pi\rangle_2 = {}& \varepsilon_{\rho\nu\alpha\beta}p_\psi^\beta q^\rho q_\mu\varepsilon_{\lambda\sigma\gamma\delta}p^\delta_{\chi_{c1}}g^{\alpha\gamma}\tilde{T}^{(1)\sigma}_{[K_1^*K]}\\ &\times\tilde{g}^{\lambda\xi}(p_{K_1^*})[\tilde{t}^{(1)}_{(14)\xi}f^{(K_1^*)}_{(134)}f^{(K^*)}_{(14)} + \{1\leftrightarrow 2, 3\leftrightarrow 4\}],\end{aligned}\tag{112}$$

$$\begin{aligned}\langle K_1^*K|K_0^*\pi\rangle_1 = {}& \varepsilon_{\mu\nu\alpha\beta}p_\psi^\beta\varepsilon_{\lambda\sigma\gamma\delta}p^\delta_{\chi_{c1}}g^{\alpha\gamma}\tilde{T}^{(1)\sigma}_{[K_1^*K]}\tilde{g}^{\lambda\xi}(p_{K_1^*})\\ &\times\tilde{t}^{(1)}_{[K_0^*\pi]\xi}[f^{(K_1^*)}_{(134)}f^{(K^*)}_{(14)} + \{1\leftrightarrow 2, 3\leftrightarrow 4\}],\end{aligned}\tag{113}$$

$$\begin{aligned}\langle K_1^*K|K_0^*\pi\rangle_2 = {}& \varepsilon_{\rho\nu\alpha\beta}p_\psi^\beta q^\rho q_\mu\varepsilon_{\lambda\sigma\gamma\delta}p^\delta_{\chi_{c1}}g^{\alpha\gamma}\tilde{T}^{(1)\sigma}_{[K_1^*K]}\\ &\times\tilde{g}^{\lambda\xi}(p_{K_1^*})\tilde{t}^{(1)}_{[K_0^*\pi]\xi}[f^{(K_1^*)}_{(134)}f^{(K^*)}_{(14)} + \{1\leftrightarrow 2, 3\leftrightarrow 4\}],\end{aligned}\tag{114}$$

$$\begin{aligned}\langle K_2^*K|K^*\pi\rangle_1 = {}& \varepsilon_{\mu\nu\alpha\beta}p_\psi^\beta\varepsilon^{\gamma\xi\delta\tau}p_{\chi_{c1}\tau}\tilde{g}^{\alpha\lambda}(p_{\chi_{c1}})P^{(2)}_{\lambda\sigma\gamma\xi}(p_{K_2^*})\\ &\times\tilde{T}^{(1)\sigma}_{[K_2^*K]}\tilde{T}^{(2)}_{[K^*\pi]\delta\eta}[\tilde{t}^{(1)\eta}_{(14)}f^{(K_1^*)}_{(134)}f^{(K^*)}_{(14)}\\ &+\{1\leftrightarrow 2, 3\leftrightarrow 4\}],\end{aligned}\tag{115}$$

$$\begin{aligned}\langle K_2^*K|K^*\pi\rangle_2 = {}& \varepsilon_{\rho\nu\alpha\beta}p_\psi^\beta q^\rho q_\mu\varepsilon^{\gamma\xi\delta\tau}p_{\chi_{c1}\tau}\tilde{g}^{\alpha\lambda}(p_{\chi_{c1}})\\ &\times P^{(2)}_{\lambda\sigma\gamma\xi}(p_{K_2^*})\tilde{T}^{(1)\sigma}_{[K_2^*K]}\tilde{T}^{(2)}_{[K^*\pi]\delta\eta}[\tilde{t}^{(1)\eta}_{(14)}f^{(K_1^*)}_{(134)}f^{(K^*)}_{(14)}\\ &+\{1\leftrightarrow 2, 3\leftrightarrow 4\}],\end{aligned}\tag{116}$$

$$\begin{aligned}\langle f_0f_2|1\rangle = {}& \varepsilon_{\mu\nu\alpha\beta}p_\psi^\beta\varepsilon_{\lambda\gamma\delta\xi}p^\xi_{\chi_{c1}}g^{\alpha\delta}\tilde{T}^{(2)\gamma}_{[f_0f_2]\sigma}\tilde{t}^{(2)\lambda\sigma}_{(12)}\\ &\times f^{(f_0)}_{(34)}f^{(f_2)}_{(12)},\end{aligned}\tag{117}$$

$$\begin{aligned}\langle f_0f_2|2\rangle = {}& \varepsilon_{\rho\nu\alpha\beta}p_\psi^\beta q^\rho q_\mu\varepsilon_{\lambda\gamma\delta\xi}p^\xi_{\chi_{c1}}g^{\alpha\delta}\tilde{T}^{(2)\gamma}_{[f_0f_2]\sigma}\tilde{t}^{(2)\lambda\sigma}_{(12)}\\ &\times f^{(f_0)}_{(34)}f^{(f_2)}_{(12)},\end{aligned}\tag{118}$$

$$\begin{aligned}\langle f_2\bar{f}_2'|1\rangle = {}& \varepsilon_{\mu\nu\alpha\beta}p_\psi^\beta\varepsilon_{\lambda\gamma\delta\xi}p^\xi_{\chi_{c1}}g^{\alpha\delta}\tilde{t}^{(2)\lambda\sigma}_{(12)}\tilde{t}^{(2)\gamma}_{(34)\sigma}\\ &\times f^{(f_2)}_{(12)}f^{(f_2')}_{(34)},\end{aligned}\tag{119}$$

$$\begin{aligned}\langle f_2\bar{f}_2'|2\rangle = {}& \varepsilon_{\rho\nu\alpha\beta}p_\psi^\beta q^\rho q_\mu\varepsilon_{\lambda\gamma\delta\xi}p^\xi_{\chi_{c1}}g^{\alpha\delta}\tilde{t}^{(2)\lambda\sigma}_{(12)}\tilde{t}^{(2)\gamma}_{(34)\sigma}\\ &\times f^{(f_2)}_{(12)}f^{(f_2')}_{(34)}.\end{aligned}\tag{120}$$

5.3 $\psi \to \gamma\chi_{c2} \to \gamma K^+K^-\pi^+\pi^-$

We construct the amplitudes $U^i_{\mu\nu}$ for the channel $\psi \to \gamma\chi_{c2} \to \gamma K^+K^-\pi^+\pi^-$. The most possible intermediate states are the same as for $\psi \to \gamma\chi_{c1} \to \gamma K^+K^-\pi^+\pi^-$.

$$\langle K_0^*\bar{K}_0^*|1\rangle = g_{\mu\nu}\tilde{T}^{(2)\alpha\beta}_{[\gamma\chi_{c2}]}\tilde{T}^{(2)}_{[K_0^*\bar{K}_0^*]\alpha\beta}f^{(K_0^*)}_{(14)}f^{(\bar{K}_0^*)}_{(23)},\tag{121}$$

$$\langle K_0^*\bar{K}_0^*|2\rangle = \tilde{T}^{(2)}_{[K_0^*\bar{K}_0^*]\mu\nu}f^{(K_0^*)}_{(14)}f^{(\bar{K}_0^*)}_{(23)},\tag{122}$$

$$\langle K_0^*\bar{K}_0^*|3\rangle = \tilde{T}^{(2)\alpha}_{[\gamma\chi_{c2}]\mu}\tilde{T}^{(2)}_{[K_0^*\bar{K}_0^*]\nu\alpha}f^{(K_0^*)}_{(14)}f^{(\bar{K}_0^*)}_{(23)},\tag{123}$$

$$\begin{aligned}\langle K_0^*\bar{K}^*|1\rangle = {}& P^{(2)}_{\mu\nu\lambda\sigma}(p_{\chi_{c2}})[\tilde{t}^{(1)\sigma}_{[K_0^*\bar{K}^*]}\tilde{t}^{(1)\lambda}_{(23)}f^{(K_0^*)}_{(14)}f^{(\bar{K}^*)}_{(23)}\\ &+\{1\leftrightarrow 2, 3\leftrightarrow 4\}],\end{aligned}\tag{124}$$

$$\begin{aligned}\langle K_0^*\bar{K}^*|2\rangle = {}& P^{(2)}_{\beta\nu\lambda\sigma}(p_{\chi_{c2}})\tilde{T}^{(2)\beta}_{[\gamma\chi_{c2}]\mu}[\tilde{t}^{(1)\sigma}_{[K_0^*\bar{K}^*]}\tilde{t}^{(1)\lambda}_{(23)}\\ &\times f^{(K_0^*)}_{(14)}f^{(\bar{K}^*)}_{(23)} + \{1\leftrightarrow 2, 3\leftrightarrow 4\}],\end{aligned}\tag{125}$$

$$\langle K_0^*\bar{K}^*|3\rangle = g_{\mu\nu}P^{(2)}_{\alpha\beta\lambda\sigma}(p_{\chi_{c2}})\tilde{T}^{(2)\alpha\beta}_{[\gamma\chi_{c2}]}[\tilde{t}^{(1)\sigma}_{[K_0^*\bar{K}^*]}\tilde{t}^{(1)\lambda}_{(23)} \times f^{(K_0^*)}_{(14)}f^{(\bar{K}^*)}_{(23)} + \{1\leftrightarrow 2, 3\leftrightarrow 4\}], \quad (126)$$

$$\langle K_0^*\bar{K}_2^*|1\rangle = g_{\mu\nu}\tilde{T}^{(2)\alpha\beta}_{[\gamma\chi_{c2}]}P^{(2)}_{\alpha\beta\lambda\sigma}(p_{\chi_{c2}})\tilde{t}^{(2)\lambda\sigma}_{(23)}f^{(K_0^*)}_{(14)}f^{(\bar{K}_2^*)}_{(23)} + \{1\leftrightarrow 2, 3\leftrightarrow 4\}], \quad (127)$$

$$\langle K_0^*\bar{K}_2^*|2\rangle = P^{(2)}_{\mu\nu\lambda\sigma}(p_{\chi_{c2}})\tilde{t}^{(2)\lambda\sigma}_{(23)}f^{(K_0^*)}_{(14)}f^{(\bar{K}_2^*)}_{(23)} + \{1\leftrightarrow 2, 3\leftrightarrow 4\}], \quad (128)$$

$$\langle K_0^*\bar{K}_2^*|3\rangle = \tilde{T}^{(2)\alpha}_{[\gamma\chi_{c2}]\mu}P^{(2)}_{\nu\alpha\lambda\sigma}(p_{\chi_{c2}})\tilde{t}^{(2)\lambda\sigma}_{(23)}f^{(K_0^*)}_{(14)}f^{(\bar{K}_2^*)}_{(23)} + \{1\leftrightarrow 2, 3\leftrightarrow 4\}], \quad (129)$$

$$\langle K_2^*\bar{K}_2^*|1\rangle = g_{\mu\nu}\tilde{T}^{(2)\alpha\beta}_{[\gamma\chi_{c2}]}P^{(2)}_{\alpha\beta\lambda\sigma}(p_{\chi_{c2}})\tilde{t}^{(2)\sigma\rho}_{(14)}\tilde{t}^{(2)\lambda}_{(23)\rho} \times f^{(K_2^*)}_{(14)}f^{(\bar{K}_2^*)}_{(23)}, \quad (130)$$

$$\langle K_2^*\bar{K}_2^*|2\rangle = P^{(2)}_{\mu\nu\lambda\sigma}(p_{\chi_{c2}})\tilde{t}^{(2)\sigma\rho}_{(14)}\tilde{t}^{(2)\lambda}_{(23)\rho}f^{(K_2^*)}_{(14)}f^{(\bar{K}_2^*)}_{(23)}, \quad (131)$$

$$\langle K_2^*\bar{K}_2^*|3\rangle = \tilde{T}^{(2)\alpha}_{[\gamma\chi_{c2}]\mu}P^{(2)}_{\nu\alpha\lambda\sigma}(p_{\chi_{c2}})\tilde{t}^{(2)\sigma\rho}_{(14)}\tilde{t}^{(2)\lambda}_{(23)\rho} \times f^{(K_2^*)}_{(14)}f^{(\bar{K}_2^*)}_{(23)}, \quad (132)$$

$$\langle K^*\bar{K}^*|1\rangle = g_{\mu\nu}\tilde{T}^{(2)\alpha\beta}_{[\gamma\chi_{c2}]}P^{(2)}_{\alpha\beta\lambda\sigma}(p_{\chi_{c2}})\tilde{t}^{(1)\lambda}_{(14)}\tilde{t}^{(1)\sigma}_{(23)} \times f^{(K^*)}_{(14)}f^{(\bar{K}^*)}_{(23)}, \quad (133)$$

$$\langle K^*\bar{K}^*|2\rangle = P^{(2)}_{\mu\nu\lambda\sigma}(p_{\chi_{c2}})\tilde{t}^{(1)\lambda}_{(14)}\tilde{t}^{(1)\sigma}_{(23)}f^{(K^*)}_{(14)}f^{(\bar{K}^*)}_{(23)}, \quad (134)$$

$$\langle K^*\bar{K}^*|3\rangle = \tilde{T}^{(2)\alpha}_{[\gamma\chi_{c2}]\mu}P^{(2)}_{\nu\alpha\lambda\sigma}(p_{\chi_{c2}})\tilde{t}^{(1)\lambda}_{(14)}\tilde{t}^{(1)\sigma}_{(23)} \times f^{(K^*)}_{(14)}f^{(\bar{K}^*)}_{(23)}, \quad (135)$$

$$\langle K_1^*K|K\rho\rangle_1 = g_{\mu\nu}\tilde{T}^{(2)\alpha\beta}_{[\gamma\chi_{c2}]}P^{(2)}_{\alpha\beta\lambda\sigma}(p_{\chi_{c2}})\tilde{T}^{(1)\sigma}_{[K_1^*\bar{K}]}\tilde{g}^{\lambda\delta}(p_{K_1^*}) \times[\tilde{t}^{(1)}_{(34)\delta}f^{(K_1^*)}_{(134)}f^{(\rho)}_{(34)} + \{1\leftrightarrow 2, 3\leftrightarrow 4\}], \quad (136)$$

$$\langle K_1^*K|K\rho\rangle_2 = P^{(2)}_{\mu\nu\lambda\sigma}(p_{\chi_{c2}})\tilde{T}^{(1)\sigma}_{[K_1^*\bar{K}]}\tilde{g}^{\lambda\delta}(p_{K_1^*}) \times[\tilde{t}^{(1)}_{(34)\delta}f^{(K_1^*)}_{(134)}f^{(\rho)}_{(34)} + \{1\leftrightarrow 2, 3\leftrightarrow 4\}], \quad (137)$$

$$\langle K_1^*K|K\rho\rangle_3 = \tilde{T}^{(2)\alpha}_{[\gamma\chi_{c2}]\mu}P^{(2)}_{\nu\alpha\lambda\sigma}(p_{\chi_{c2}})\tilde{T}^{(1)\sigma}_{[K_1^*\bar{K}]}\tilde{g}^{\lambda\delta}(p_{K_1^*}) \times[\tilde{t}^{(1)}_{(34)\delta}f^{(K_1^*)}_{(134)}f^{(\rho)}_{(34)} + \{1\leftrightarrow 2, 3\leftrightarrow 4\}], \quad (138)$$

$$\langle K_1^*K|K^*\pi\rangle_1 = g_{\mu\nu}\tilde{T}^{(2)\alpha\beta}_{[\gamma\chi_{c2}]}P^{(2)}_{\alpha\beta\lambda\sigma}(p_{\chi_{c2}})\tilde{T}^{(1)\sigma}_{[K_1^*\bar{K}]}\tilde{g}^{\lambda\delta}(p_{K_1^*}) \times[\tilde{t}^{(1)}_{(14)\delta}f^{(K_1^*)}_{(134)}f^{(K^*)}_{(14)} + \{1\leftrightarrow 2, 3\leftrightarrow 4\}], \quad (139)$$

$$\langle K_1^*K|K^*\pi\rangle_2 = P^{(2)}_{\mu\nu\lambda\sigma}(p_{\chi_{c2}})\tilde{T}^{(1)\sigma}_{[K_1^*\bar{K}]}\tilde{g}^{\lambda\delta}(p_{K_1^*}) \times[\tilde{t}^{(1)}_{(14)\delta}f^{(K_1^*)}_{(134)}f^{(K^*)}_{(14)} + \{1\leftrightarrow 2, 3\leftrightarrow 4\}], \quad (140)$$

$$\langle K_1^*K|K^*\pi\rangle_3 = \tilde{T}^{(2)\alpha}_{[\gamma\chi_{c2}]\mu}P^{(2)}_{\nu\alpha\lambda\sigma}(p_{\chi_{c2}})\tilde{T}^{(1)\sigma}_{[K_1^*\bar{K}]}\tilde{g}^{\lambda\delta}(p_{K_1^*}) \times[\tilde{t}^{(1)}_{(14)\delta}f^{(K_1^*)}_{(134)}f^{(K^*)}_{(14)} + \{1\leftrightarrow 2, 3\leftrightarrow 4\}], \quad (141)$$

$$\langle K_1^*K|K_0^*\pi\rangle_1 = g_{\mu\nu}\tilde{T}^{(2)\alpha\beta}_{[\gamma\chi_{c2}]}P^{(2)}_{\alpha\beta\lambda\sigma}(p_{\chi_{c2}})\tilde{T}^{(1)\sigma}_{[K_1^*\bar{K}]}\tilde{g}^{\lambda\delta}(p_{K_1^*}) \times\tilde{t}^{(1)}_{(K_0^*\pi)\delta}[f^{(K_1^*)}_{(134)}f^{(K_0^*)}_{(14)} + \{1\leftrightarrow 2, 3\leftrightarrow 4\}], \quad (142)$$

$$\langle K_1^*K|K_0^*\pi\rangle_2 = P^{(2)}_{\mu\nu\lambda\sigma}(p_{\chi_{c2}})\tilde{T}^{(1)\sigma}_{[K_1^*\bar{K}]}\tilde{g}^{\lambda\delta}(p_{K_1^*})\tilde{t}^{(1)}_{(K_0^*\pi)\delta} \times[f^{(K_1^*)}_{(134)}f^{(K_0^*)}_{(14)} + \{1\leftrightarrow 2, 3\leftrightarrow 4\}], \quad (143)$$

$$\langle K_1^*K|K_0^*\pi\rangle_3 = \tilde{T}^{(2)\alpha}_{[\gamma\chi_{c2}]\mu}P^{(2)}_{\nu\alpha\lambda\sigma}(p_{\chi_{c2}})\tilde{T}^{(1)\sigma}_{[K_1^*\bar{K}]}\tilde{g}^{\lambda\delta}(p_{K_1^*})\tilde{t}^{(1)}_{(K_0^*\pi)\delta} \times[f^{(K_1^*)}_{(134)}f^{(K_0^*)}_{(14)} + \{1\leftrightarrow 2, 3\leftrightarrow 4\}], \quad (144)$$

$$\langle K_2^*K|K^*\pi\rangle_1 = g_{\mu\nu}\tilde{T}^{(2)\alpha\beta}_{[\gamma\chi_{c2}]}P^{(2)}_{\alpha\beta\lambda\xi}(p_{\chi_{c2}})P^{(2)\xi\sigma\xi'\sigma'}(p_{K_2^*})\varepsilon^{\lambda\gamma\delta}_{\sigma} \times p_{\chi_{c2}\delta}\varepsilon^{\gamma'\eta'\delta'}_{\sigma'}p_{K_2^*\delta'}\tilde{T}^{(1)}_{[K_2^*\bar{K}]\gamma}\tilde{t}^{(2)}_{(K^*\pi)\gamma'\xi'}[\tilde{t}^{(1)}_{(14)\eta'} \times f^{(K_1^*)}_{(134)}f^{(K^*)}_{(14)} + \{1\leftrightarrow 2, 3\leftrightarrow 4\}], \quad (145)$$

$$\langle K_2^*K|K^*\pi\rangle_2 = P^{(2)}_{\mu\nu\lambda\xi}(p_{\chi_{c2}})P^{(2)\xi\sigma\xi'\sigma'}(p_{K_2^*})\varepsilon^{\lambda\gamma\delta}_{\sigma}p_{\chi_{c2}\delta}\varepsilon^{\gamma'\eta'\delta'}_{\sigma'} \times p_{K_2^*\delta'}\tilde{T}^{(1)}_{[K_2^*\bar{K}]\gamma}\tilde{t}^{(2)}_{(K^*\pi)\gamma'\xi'}[\tilde{t}^{(1)}_{(14)\eta'} \times f^{(K_1^*)}_{(134)}f^{(K^*)}_{(14)} + \{1\leftrightarrow 2, 3\leftrightarrow 4\}], \quad (146)$$

$$\langle K_2^*K|K^*\pi\rangle_3 = \tilde{T}^{(2)\alpha}_{[\gamma\chi_{c2}]\mu}P^{(2)}_{\nu\alpha\lambda\xi}(p_{\chi_{c2}})P^{(2)\xi\sigma\xi'\sigma'}(p_{K_2^*})\varepsilon^{\lambda\gamma\delta}_{\sigma} \times p_{\chi_{c2}\delta}\varepsilon^{\gamma'\eta'\delta'}_{\sigma'}p_{K_2^*\delta'}\tilde{T}^{(1)}_{[K_2^*\bar{K}]\gamma}\tilde{t}^{(2)}_{(K^*\pi)\gamma'\xi'} \times[\tilde{t}^{(1)}_{(14)\eta'}f^{(K_1^*)}_{(134)}f^{(K^*)}_{(14)} + \{1\leftrightarrow 2, 3\leftrightarrow 4\}], \quad (147)$$

$$\langle f_0f_0'|1\rangle = g_{\mu\nu}\tilde{T}^{(2)\alpha\beta}_{[\gamma\chi_{c2}]}\tilde{T}^{(2)}_{[f_0f_0']\alpha\beta}f^{(f_0')}_{(12)}f^{(f_0)}_{(34)}, \quad (148)$$

$$\langle f_0f_0'|2\rangle = \tilde{T}^{(2)}_{[f_0f_0']\mu\nu}f^{(f_0')}_{(12)}f^{(f_0)}_{(34)}, \quad (149)$$

$$\langle f_0f_0'|3\rangle = \tilde{T}^{(2)\alpha}_{[\gamma\chi_{c2}]\mu}\tilde{T}^{(2)}_{[f_0f_0']\nu\alpha}f^{(f_0')}_{(12)}f^{(f_0)}_{(34)}, \quad (150)$$

$$\langle f_0f_2|1\rangle = g_{\mu\nu}\tilde{T}^{(2)\alpha\beta}_{[\gamma\chi_{c2}]}P^{(2)}_{\alpha\beta\lambda\sigma}(p_{\chi_{c2}})\tilde{t}^{(2)\lambda\sigma}_{(12)} \times f^{(f_2)}_{(12)}f^{(f_0)}_{(34)}, \quad (151)$$

$$\langle f_0f_2|2\rangle = P^{(2)}_{\mu\nu\lambda\sigma}(p_{\chi_{c2}})\tilde{t}^{(2)\lambda\sigma}_{(12)}f^{(f_2)}_{(12)}f^{(f_0)}_{(34)}, \quad (152)$$

$$\langle f_0f_2|3\rangle = \tilde{T}^{(2)\alpha}_{[\gamma\chi_{c2}]\mu}P^{(2)}_{\nu\alpha\lambda\sigma}(p_{\chi_{c2}})\tilde{t}^{(2)\lambda\sigma}_{(12)}f^{(f_2)}_{(12)}f^{(f_0)}_{(34)}, \quad (153)$$

$$\langle f_2f_2'|1\rangle = g_{\mu\nu}\tilde{T}^{(2)\alpha\beta}_{[\gamma\chi_{c2}]}P^{(2)}_{\alpha\beta\lambda\sigma}(p_{\chi_{c2}})\tilde{t}^{(2)\sigma\rho}_{(12)} \times\tilde{t}^{(2)\lambda}_{(34)\rho}f^{(f_2)}_{(12)}f^{(f_2')}_{(34)}, \quad (154)$$

$$\langle f_2f_2'|2\rangle = P^{(2)}_{\mu\nu\lambda\sigma}(p_{\chi_{c2}})\tilde{t}^{(2)\sigma\rho}_{(12)}\tilde{t}^{(2)\lambda}_{(34)\rho}f^{(f_2)}_{(12)}f^{(f_2')}_{(34)}, \quad (155)$$

$$\langle f_2f_2'|3\rangle = \tilde{T}^{(2)\alpha}_{[\gamma\chi_{c2}]\mu}P^{(2)}_{\nu\alpha\lambda\sigma}(p_{\chi_{c2}})\tilde{t}^{(2)\sigma\rho}_{(12)}\tilde{t}^{(2)\lambda}_{(34)\rho} \times\tilde{t}^{(2)}_{(34)\eta\tau}f^{(f_2)}_{(12)}f^{(f_2')}_{(34)}. \quad (156)$$

5.4 $\psi \to \gamma\chi_{c0} \to \gamma\pi^+\pi^-\pi^+\pi^-$

We construct the amplitudes $U^i_{\mu\nu}$ with a notation similar to the $\psi \to \gamma\chi_{c0} \to \gamma K^+K^-\pi^+\pi^-$ channel. Here we denote π^+, π^-, π^+, π^- as 1, 2, 3, 4.

$$\langle f_0f_0|1\rangle = g_{\mu\nu}[f^{(f_0)}_{(12)}f^{(f_0)}_{(34)} + \{2\leftrightarrow 4\}], \quad (157)$$

$$\langle f_0f_2|1\rangle = g_{\mu\nu}[\tilde{T}^{(2)\alpha\beta}_{[f_0^{(12)}f_2^{(34)}]}\tilde{t}^{(2)}_{(34)\alpha\beta}f^{(f_0)}_{(12)}f^{(f_2)}_{(34)} + \{1\leftrightarrow 3\} + \{2\leftrightarrow 4\} + \{1\leftrightarrow 3, 2\leftrightarrow 4\}], \quad (158)$$

$$\langle f_2f_2|1\rangle = g_{\mu\nu}[f^{(f_2)}_{(12)}f^{(f_2)}_{(34)}\tilde{t}^{(2)\alpha\beta}_{(12)}\tilde{t}^{(2)}_{(34)\alpha\beta} + \{2\leftrightarrow 4\}], \quad (159)$$

$$\langle f_2f_2|2\rangle = g_{\mu\nu}[f^{(f_2)}_{(12)}f^{(f_2)}_{(34)}\tilde{T}^{(2)\alpha\beta}_{[f_2^{(12)}f_2^{(34)}]}\tilde{t}^{(2)\gamma}_{(12)\alpha}\tilde{t}^{(2)}_{(34)\beta\gamma} + \{2\leftrightarrow 4\}], \quad (160)$$

$$\langle\rho\rho|1\rangle = g_{\mu\nu}[f^{(\rho)}_{(12)} f^{(\rho)}_{(34)} \tilde{t}^{(1)\alpha}_{(12)} \tilde{t}^{(1)}_{(34)\alpha} + \{2 \leftrightarrow 4\}], \tag{161}$$

$$\langle\rho\rho|2\rangle = g_{\mu\nu}[f^{(\rho)}_{(12)} f^{(\rho)}_{(34)} \tilde{T}^{(2)\alpha\beta}_{[\rho(12)\rho(34)]} \tilde{t}^{(1)}_{(12)\alpha} \tilde{t}^{(1)}_{(34)\beta} + \{2 \leftrightarrow 4\}], \tag{162}$$

$$\langle\pi\pi'|\pi\sigma\rangle = g_{\mu\nu}[f^{(\pi')}_{(123)}(f^{(\sigma)}_{(12)} + f^{(\sigma)}_{(32)}) + f^{(\pi')}_{(234)}(f^{(\sigma)}_{(23)} + f^{(\sigma)}_{(34)}) + f^{(\pi')}_{(143)}(f^{(\sigma)}_{(14)} + f^{(\sigma)}_{(34)}) + f^{(\pi')}_{(214)}(f^{(\sigma)}_{(21)} + f^{(\sigma)}_{(14)})], \tag{163}$$

$$\langle\pi\pi'|\pi\rho\rangle = g_{\mu\nu}[f^{(\pi')}_{(123)} f^{(\rho)}_{(12)} \tilde{t}^{(1)\alpha}_{(\rho 3)} \tilde{t}^{(1)}_{(12)\alpha} + f^{(\pi')}_{(234)} f^{(\rho)}_{(23)} \tilde{t}^{(1)\alpha}_{(\rho 4)} \tilde{t}^{(1)}_{(23)\alpha} + \{1 \leftrightarrow 3\} + \{2 \leftrightarrow 4\} + \{1 \leftrightarrow 3, 2 \leftrightarrow 4\}], \tag{164}$$

$$\langle\pi a_1|\pi\sigma\rangle = g_{\mu\nu}[f^{(a_1)}_{(123)} f^{(\sigma)}_{(12)} \tilde{T}^{(1)\alpha}_{(a_1 4)} \tilde{t}^{(1)}_{(\sigma 3)\alpha} + f^{(a_1)}_{(234)} f^{(\sigma)}_{(23)} \tilde{T}^{(1)\alpha}_{(a_1 1)} \tilde{t}^{(1)}_{(\sigma 4)\alpha} + \{1 \leftrightarrow 3\} + \{2 \leftrightarrow 4\} + \{1 \leftrightarrow 3, 2 \leftrightarrow 4\}], \tag{165}$$

$$\langle\pi a_1|\pi\rho\rangle = g_{\mu\nu}[P^{(1)}_{\alpha\beta}(p_{(123)}) \tilde{T}^{(1)\alpha}_{(a_1 4)} \tilde{t}^{(1)\beta}_{(12)} f^{(a_1)}_{(123)} f^{(\rho)}_{(12)} + P^{(1)}_{\alpha\beta}(p_{(234)}) \tilde{T}^{(1)\alpha}_{(a_1 1)} \tilde{t}^{(1)\beta}_{(23)} f^{(a_1)}_{(234)} f^{(\rho)}_{(23)} + \{1 \leftrightarrow 3\} + \{2 \leftrightarrow 4\} + \{1 \leftrightarrow 3, 2 \leftrightarrow 4\}]. \tag{166}$$

5.5 $\psi \rightarrow \gamma\chi_{c1} \rightarrow \gamma\pi^+\pi^-\pi^+\pi^-$

In this subsection we construct the amplitudes $U^i_{\mu\nu}$ for the process $\psi \rightarrow \gamma\chi_{c1} \rightarrow \gamma\pi^+\pi^-\pi^+\pi^-$. The most possible intermediate states are: $f_0 f_2$, $f_2 f_2$, and $\rho\rho$ with f_0, f_2, and $\rho \rightarrow \pi^+\pi^-$.

$$\langle f_0 f_2|1\rangle = \varepsilon_{\mu\nu\alpha\beta} p^\beta_\psi \varepsilon_{\lambda\gamma\delta\xi} p^\xi_{\chi_{c1}} g^{\alpha\delta}[\tilde{T}^{(2)\gamma}_{[f_0 \bar{f}_2]\sigma} \tilde{t}^{(2)\lambda\sigma}_{(12)} f^{(f_0)}_{(34)} f^{(f_2)}_{(12)} + \{1 \leftrightarrow 3\} + \{2 \leftrightarrow 4\} + \{1 \leftrightarrow 3, 2 \leftrightarrow 4\}], \tag{167}$$

$$\langle f_0 f_2|2\rangle = \varepsilon_{\rho\nu\alpha\beta} p^\beta_\psi q^\rho q_\mu \varepsilon_{\lambda\gamma\delta\xi} p^\xi_{\chi_{c1}} g^{\alpha\delta}[\tilde{T}^{(2)\gamma}_{[f_0 \bar{f}_2]\sigma} \tilde{t}^{(2)\lambda\sigma}_{(12)} \times f^{(f_0)}_{(34)} f^{(f_2)}_{(12)} + \{1 \leftrightarrow 3\} + \{2 \leftrightarrow 4\} + \{1 \leftrightarrow 3, 2 \leftrightarrow 4\}], \tag{168}$$

$$\langle f_2 f_2|1\rangle = \varepsilon_{\mu\nu\alpha\beta} p^\beta_\psi \varepsilon_{\lambda\gamma\delta\xi} p^\xi_{\chi_{c1}} g^{\alpha\delta}[\tilde{t}^{(2)\lambda\sigma}_{(12)} \tilde{t}^{(2)\gamma}_{(34)\sigma} f^{(f_2)}_{(12)} f^{(f_2)}_{(34)} + \{2 \leftrightarrow 4\}], \tag{169}$$

$$\langle f_2 f_2|2\rangle = \varepsilon_{\rho\nu\alpha\beta} p^\beta_\psi q^\rho q_\mu \varepsilon_{\lambda\gamma\delta\xi} p^\xi_{\chi_{c1}} g^{\alpha\delta}[\tilde{t}^{(2)\lambda\sigma}_{(12)} \tilde{t}^{(2)\gamma}_{(34)\sigma} f^{(K^*_2)}_{(12)} f^{(f_2)}_{(34)} + \{2 \leftrightarrow 4\}], \tag{170}$$

$$\langle\rho\rho|1\rangle = \varepsilon_{\mu\nu\alpha\beta} p^\beta_\psi \varepsilon_{\lambda\sigma\gamma\delta} p^\delta_{\chi_{c1}} g^{\alpha\gamma}[\tilde{t}^{(1)\lambda}_{(12)} \tilde{t}^{(1)\sigma}_{(34)} f^{(\rho)}_{(12)} f^{(\rho)}_{(34)} + \{2 \leftrightarrow 4\}], \tag{171}$$

$$\langle\rho\rho|2\rangle = \varepsilon_{\xi\nu\alpha\beta} p^\beta_\psi q^\xi q_\mu \varepsilon_{\lambda\sigma\gamma\delta} p^\delta_{\chi_{c1}} g^{\alpha\gamma}[\tilde{t}^{(1)\lambda}_{(12)} \tilde{t}^{(1)\sigma}_{(34)} f^{(\rho)}_{(12)} f^{(\rho)}_{(34)} + \{2 \leftrightarrow 4\}]. \tag{172}$$

5.6 $\psi \rightarrow \gamma\chi_{c2} \rightarrow \gamma\pi^+\pi^-\pi^+\pi^-$

The most possible intermediate states are $f_0 f_0$, $f_0 f_2$, $f_2 f_2$, and $\rho\rho$ with f_0, f_2, $\rho \rightarrow \pi^+\pi^-$. Then we have the following convariant tensor amplitudes:

$$\langle f_0 f_0|1\rangle = g_{\mu\nu} \tilde{T}^{(2)\alpha\beta}_{[\gamma\chi_{c2}]} \tilde{T}^{(2)}_{[f_0 f_0]\alpha\beta}[f^{(f_0)}_{(12)} f^{(f_0)}_{(34)} + \{2 \leftrightarrow 4\}], \tag{173}$$

$$\langle f_0 f_0|2\rangle = \tilde{T}^{(2)}_{[f_0 f_0]\mu\nu}[f^{(f_0)}_{(12)} f^{(f_0)}_{(34)} + \{2 \leftrightarrow 4\}], \tag{174}$$

$$\langle f_0 f_0|3\rangle = \tilde{T}^{(2)\alpha}_{[\gamma\chi_{c2}]\mu} \tilde{T}^{(2)}_{[f_0 f_0]\nu\alpha}[f^{(f_0)}_{(12)} f^{(f_0)}_{(34)} + \{2 \leftrightarrow 4\}], \tag{175}$$

$$\langle f_0 f_2|1\rangle = g_{\mu\nu} \tilde{T}^{(2)\alpha\beta}_{[\gamma\chi_{c2}]} P^{(2)}_{\alpha\beta\lambda\sigma}(p_{\chi_{c2}})[\tilde{t}^{(2)\lambda\sigma}_{(12)} f^{(f_2)}_{(12)} f^{(f_0)}_{(34)} + \{1 \leftrightarrow 3\} + \{2 \leftrightarrow 4\} + \{1 \leftrightarrow 3, 2 \leftrightarrow 4\}], \tag{176}$$

$$\langle f_0 f_2|2\rangle = P^{(2)}_{\mu\nu\lambda\sigma}(p_{\chi_{c2}})[\tilde{t}^{(2)\lambda\sigma}_{(12)} f^{(f_2)}_{(12)} f^{(f_0)}_{(34)} + \{1 \leftrightarrow 3\} + \{2 \leftrightarrow 4\} + \{1 \leftrightarrow 3, 2 \leftrightarrow 4\}], \tag{177}$$

$$\langle f_0 f_2|3\rangle = \tilde{T}^{(2)\alpha}_{[\gamma\chi_{c2}]\mu} P^{(2)}_{\nu\alpha\lambda\sigma}(p_{\chi_{c2}})[\tilde{t}^{(2)\lambda\sigma}_{(12)} f^{(f_2)}_{(12)} f^{(f_0)}_{(34)} + \{1 \leftrightarrow 3\} + \{2 \leftrightarrow 4\} + \{1 \leftrightarrow 3, 2 \leftrightarrow 4\}], \tag{178}$$

$$\langle f_2 f_2|1\rangle = g_{\mu\nu} \tilde{T}^{(2)\alpha\beta}_{[\gamma\chi_{c2}]} P^{(2)}_{\alpha\beta\lambda\sigma}(p_{\chi_{c2}})[\tilde{t}^{(2)\sigma\rho}_{(12)} \tilde{t}^{(2)\lambda}_{(34)\rho} f^{(f_2)}_{(12)} f^{(f_2)}_{(34)} + \{2 \leftrightarrow 4\}], \tag{179}$$

$$\langle f_2 f_2|2\rangle = P^{(2)}_{\mu\nu\lambda\sigma}(p_{\chi_{c2}})[\tilde{t}^{(2)\sigma\rho}_{(12)} \tilde{t}^{(2)\lambda}_{(34)\rho} f^{(f_2)}_{(12)} f^{(f_2)}_{(34)} + \{2 \leftrightarrow 4\}], \tag{180}$$

$$\langle f_2 f_2|3\rangle = \tilde{T}^{(2)\alpha}_{[\gamma\chi_{c2}]\mu} P^{(2)}_{\nu\alpha\lambda\sigma}(p_{\chi_{c2}})[\tilde{t}^{(2)\sigma\rho}_{(12)} \tilde{t}^{(2)\lambda}_{(34)\rho} f^{(f_2)}_{(12)} f^{(f_2)}_{(34)} + \{2 \leftrightarrow 4\}], \tag{181}$$

$$\langle\rho\rho|1\rangle = g_{\mu\nu} \tilde{T}^{(2)\alpha\beta}_{[\gamma\chi_{c2}]} P^{(2)}_{\alpha\beta\lambda\sigma}(p_{\chi_{c2}})[\tilde{t}^{(1)\lambda}_{(12)} \tilde{t}^{(1)\sigma}_{(34)} f^{(\rho)}_{(12)} f^{(\rho)}_{(34)} + \{2 \leftrightarrow 4\}], \tag{182}$$

$$\langle\rho\rho|2\rangle = P^{(2)}_{\mu\nu\lambda\sigma}(p_{\chi_{c2}})[\tilde{t}^{(1)\lambda}_{(12)} \tilde{t}^{(1)\sigma}_{(34)} f^{(\rho)}_{(12)} f^{(\rho)}_{(34)} + \{2 \leftrightarrow 4\}], \tag{183}$$

$$\langle\rho\rho|3\rangle = \tilde{T}^{(2)\alpha}_{[\gamma\chi_{c2}]\mu} P^{(2)}_{\nu\alpha\lambda\sigma}(p_{\chi_{c2}})[\tilde{t}^{(1)\lambda}_{(12)} \tilde{t}^{(1)\sigma}_{(34)} f^{(\rho)}_{(12)} f^{(\rho)}_{(34)} + \{2 \leftrightarrow 4\}]. \tag{184}$$

Here f_0, f_2 and ρ can be replaced by any f'_0, f'_2 and ρ', respectively.

6 Conclusion

First of all, we provide a theoretical PWA formalism for the radiative decay $J/\psi \rightarrow \gamma p\bar{p}$, which is also applicable to the processes $J/\psi \rightarrow \gamma\Lambda\bar{\Lambda}, \gamma\Sigma\bar{\Sigma}$ and $\gamma\Xi\bar{\Xi}$. Then we present a general covariant formalism for the PWA of the double radiative decay $\psi \rightarrow \gamma\gamma V(\rho,\omega,\phi)$ processes. Finally, we give the PWA formulae for $\psi(2s)$ radiative decays into $K^+K^-\pi^+\pi^-$ and $\pi^+\pi^-\pi^+\pi^-$ that are very useful to study χ_{cJ} charmonium states. We have constructed most possible covariant tensor amplitudes for intermediate resonant states of $J \leq 2$. For intermediate resonant states of $J \geq 3$, the production vertices need $L \geq 2$ and are expected to be suppressed [11]. The formulae here can be directly used to perform partial-wave analysis of forthcoming high statistics data from CLEO-c and BES-III on these channels to extract useful information on the baryon-antibaryon interactions, and $\psi \rightarrow \gamma\gamma V(\rho,\omega,\phi)$ processes to extract information on the flavor content of any meson resonances (R) with positive charge parity ($C = +$) and mass above 1 GeV, as well as $\psi(2s) \rightarrow \gamma\chi_{cJ}$ with χ_{cJ}

decays into $K^+K^-\pi^+\pi^-$ and $\pi^+\pi^-\pi^+\pi^-$ to study gluon hadronization dynamics.

We thank Z.J. Guo and C.Z. Yuan for useful discussions. The work of S. Dulat is supported by the National Natural Sciences Foundation of China under Grant No. 10465004, and partly by the Abdus Salam International Centre for Theoretical Physics, Trieste, Italy, as well as by Xinjiang University. The work of B.S. Zou is partly supported by CAS Knowledge Innovation Project (KJCX2-SW-N02) and the National Natural Sciences Foundation of China under Grant Nos. 10225525,10435080.

References

1. BES Collaboration, Phys. Rev. Lett. **91**, 022001, (2003).
2. For example, B.S. Zou, H.C. Chiang, Phys. Rev. D **69**, 034004 (2004); B. Kerbikov, A. Stavinsky, V. Fedotov, Phys. Rev. C **69**, 055205 (2004); A. Sibirtsev *et al.*, Phys. Rev. D **71**, 054010 (2005); B. Loiseau, S. Wycech, Int. J. Mod. Phys. A **20**, 1990 (2005).
3. F.E. Close, RAL-84-095, hep-ph/0311087; F.E. Close, A Donnachie, Y. Kalashnikova, Phys. Rev. D **67**, 074031 (2003); B.S. Zou, Nucl. Phys. A **692**, 362 (2001).
4. C. Edwards, PhD Thesis, Caltech preprint CALT-68-1165 (1985).
5. J.E. Augustin *et al.*, Orsay preprint LAL/85-27 (1985).
6. D. Coffman *et al.*, Phys. Rev. D **41**, 1410 (1990).
7. BES Collaboration, Nucl. Phys. A **675**, 337 (2000).
8. W. Rarita, J. Schwinger, Phys. Rev. **60**, 61 (1941);
9. S.U. Chung, Phys. Rev. D **48**, 1225 (1993); **57**, 431 (1998).
10. V. Filippini, A. Fontana, A. Rotondi, Phys. Rev. D **51**, 2247 (1995).
11. B.S. Zou, D.V. Bugg, Eur. Phys. J. A **16**, 537 (2003).
12. BES Collaboration, Phys. Lett. B **440**, 217 (1998); **446**, 356 (1999); **472**, 200 (2000); **472**, 208 (2000); **476**, 25 (2000); Phys. Rev. D **68**, 052003 (2003); Phys. Lett. B **598**, 149 (2004); **603**, 138 (2004); **607**, 243 (2005).
13. V.V. Anisovich, D.V. Bugg, A.V. Sarantsev, Nucl. Phys. A **537**, 501 (1992); V.V. Anisovich *et al.*, Nucl. Phys. A **544**, 747 (1992).
14. A.V. Anisovich *et al.*, Phys. Lett. B **452**, 173; 180; 187 (1999); **468**, 304; 309 (1999); **500**, 222 (2001); **507**, 23 (2001).
15. A.V. Anisovich, V.A. Sadovnikova, Eur. Phys. J. A **2**, 199 (1998); A.V. Anisovich *et al.*, J. Phys. G **28**, 15 (2002).
16. Sayipjamal Dulat, Bing-Song Zou, Ji-Min Wu, High Energy Phys. Nucl. Phys. **28**, 350 (2004).
17. F. von Hippel, C. Quigg, Phys. Rev. D **5**, 624 (1972).
18. W. Greiner *et al.*, *Quantum Chromodynamics* (Springer, 2002) p. 242.
19. B.S. Zou, F. Hussain, Phys. Rev. C **67**, 015204 (2003).